Working Paper Series No. 06/2006

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Inflation Differentials in a Currency Union: A DSGE Perspective

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December 2006

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"la Caixa"

December 29, 2006

Abstract

Cross country studies of inflation differentials, in particular in the EMU, have focused on three explanations: (i) the role of tradable and nontradable sector technology shocks and the Balassa-Samuelson effect, (ii) the role of the demand-side effects, and (iii) heterogeneity of inflationary processes inside the EMU. This paper estimates a two country, two sector Dynamic Stochastic General Equilibrium (DSGE) model with nominal rigidities in a currency union using data for Spain and the euro area, to understand the role of each feature in shaping inflation differentials. The paper finds that tradable sector technology shocks are the most important source of inflation differentials, while nontradable sector technology shocks help explain nontradable inflation only, and demand shocks help explain a fraction of output growth, but not of inflation dispersion. In addition, the estimated model finds evidence against inflation dynamics being different in Spain and in the rest of the euro area.

JEL Classification: F41, F42, C51.

Keywords: Balassa-Samuelson effect, Bayesian Estimation, European Monetary Union.

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1 Introduction

Since the launch of the common european currency, the euro, in January 1999, a topic that has received a lot of attention is the study of inflation differentials in the European Monetary Union (EMU).[1] At the time the euro and the common monetary policy were introduced, the Harmonised Index of Consumer Prices (HICP) was increasing at a 12-month rate of 0.9 percent for the EMU, with a weighted standard deviation of 1.1 percent. Seven years later, in January 2006, the EMU inflation rate was at 2.4 percent, while the weighted standard deviation was 2.6 percent. This increase both in inflation and inflation dispersion can be striking given that in January 1999, EMU countries seemed to have achieved nominal convergence. Figure 1 plots the weighted standard deviation of the 12-month inflation rate, and its components (goods and services). Clearly, after an all-time low in 1999, inflation dispersion has increased significantly since, albeit with some fluctuations. While most of the time there has been higher dispersion in nontraded (services) inflation, in two episodes (between early 2000 to mid-2001, and since late-2005) the opposite has happened, and the traded goods component (the "goods" category in the HICP) has in fact displayed more dispersion across EMU countries.

Another interesting feature of the EMU is the persistence of inflation differentials. Even when long periods of time are considered, some member countries have persistently experienced higher inflation rates than the EMU as a whole. Table 1 shows the average 12-month HIPC inflation rates for the period January 1999 - July 2006 for the 12 countries of the EMU. While EMU has been on average right above the ECB's target of 2 percent inflation, there are some important cross-country differences. Some countries have been, on average, close or below the ECB target (Austria, France, Belgium, Finland and Germany); while, on the other hand, some countries have been significantly above the target: well known examples in this last group are Spain, with a seven-year average 12-month inflation rate of 3.2 percent, and Ireland, with 3.5 percent. Table 1 also shows that inflation in the services component of the HICP has been higher than in the goods component, and that the national pattern that we observe for the headline HICP also holds for its goods, services, and core (excluding food and energy) components.

[1] See for instance ECB (2003), Angeloni and Erhmann (2004), López-Salido et al. (2005), Andrés et al. (2003).

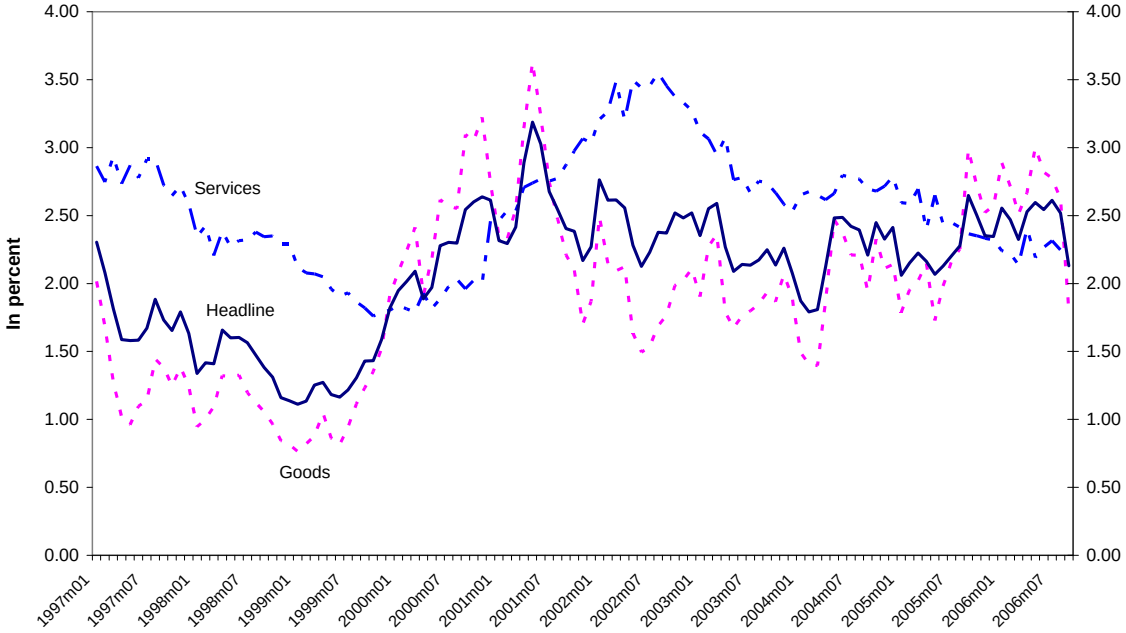

Figure 1: Weighted standard deviation of the 12-month inflation rate in the EMU. Source: Eurostat and author's calculations

Table 1. Average 12-month HICP inflation rates. Euro area, 1999-2006

	HICP	Goods	Services	Core
EMU	2.07	1.92	2.28	1.71
Belgium (BE)	2.01	1.93	2.12	1.55
Germany (DE)	1.48	1.59	1.35	0.97
Greece (GR)	3.24	2.98	3.70	3.08
Spain (ES)	3.17	2.84	3.78	2.86
France (FR)	1.81	1.64	2.07	1.53
Ireland (IE)	3.50	2.37	5.01	3.36
Italy (IT)	2.35	2.12	2.69	2.19
Luxemburg (LU)	2.79	2.79	2.72	2.33
Netherlands (NL)	2.57	2.20	3.08	2.09
Austria (AT)	1.70	1.27	2.22	1.46
Portugal (PT)	3.02	2.37	4.05	2.88
Finland (FI)	1.58	1.05	2.46	1.35

Source: Eurostat and author's calculations

Cross country studies of inflation dynamics, and in particular in the EMU, have focused on three main explanations. The first one brings back the well-known Balassa-Samuelson effect. The second one studies the role of the demand-side effects as well as the asymmetric position of the business cycle in the economies of a currency union. The third one studies heterogeneity of inflationary processes inside the EMU, which could make inflation differentials highly persistent, even when all countries are hit by the same symmetric shocks (for instance, oil prices, or fluctuations of the euro).

The Balassa-Samuelson effect is typically used to explain inflation differentials for those countries experiencing a catching-up process. As the relatively poorer countries adopt new technologies and get closer to the most advanced countries, they will necessarily experience higher real GDP growth, increased wages, and higher inflation. The Balassa-Samuelson effect can be stated as follows: suppose that the sectors of an economy that are open to international trade (the "tradable" sectors) experience high productivity growth. This can happen, as in the case of the EMU, when a group of countries increase economic integration, barriers to trade fall, and hence it is easier to import more productive technologies from the more advanced countries. The higher productivity in the tradable sector increases the marginal product of labor in that sector, and therefore labor demand. This puts upward pressure on wages, which increase for the whole economy. Since prices are set as a markup over production costs, inflation increases in the sectors of the economy not open to international trade (the "nontradable" sector), that do not benefit from productivity improvements but face higher wages. The effect of productivity improvements on tradable inflation in the short term is less clear, but typically the real wage increases by less than the level of productivity, and tradable inflation declines. Therefore, the Balassa-Samuelson hypothesis could be a candidate to explain the higher inflation rate in the service sector (that does not benefit from productivity improvements) than in the goods sector, and hence leading to higher headline HICP inflation.[2]

At first sight, this story seems to fit the EMU experience: Spain and Ireland, for instance, have experienced above-average real GDP growth and above-average inflation. In Spain, labor productivity growth has been much higher in the tradable sector than in the nontradable sector. Figure 2 plots labor productivity in the

[2] Regarding inflation differentials in the tradable sector, as trade barriers fall and countries adopt a common currency (hence, price comparisons are easier), then price level convergence implies that some countries will experience higher inflation rates than others in the transition. However, Rogers (2006) finds that price level convergence in the EMU seemed to happen already during the 1990s, and that current levels of price dispersion across european cities are similar to those in the USA.

two sectors (defined as output per employee). In fact, productivity in the nontradable sector (that includes services and construction) has been experiencing negative growth rates in recent years.

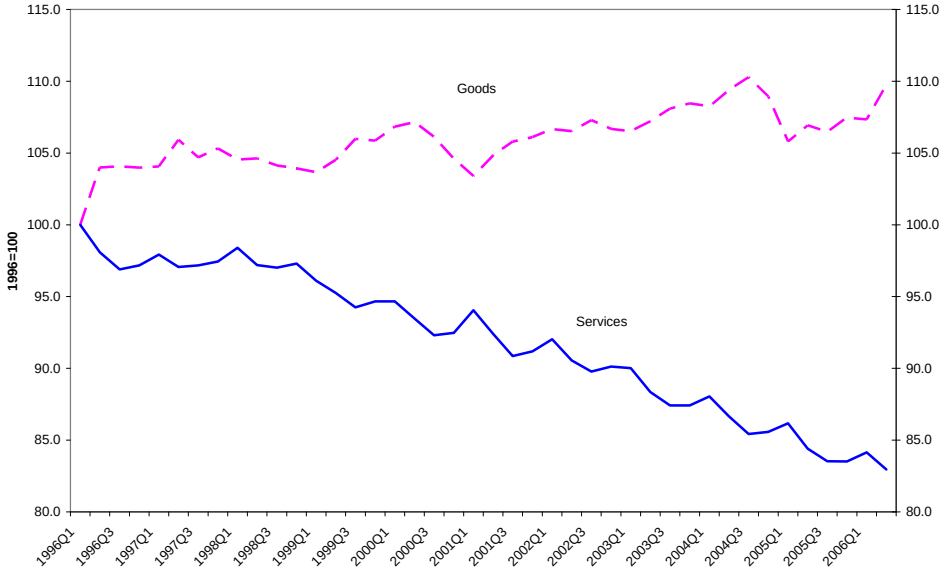

Figure 2: Spain. Labor productivity in the tradable (goods) and nontradable (services and construction) sectors. Source: INE.

As López-Salido et al. (2005) point out, it is difficult to square the evidence on productivity and inflation with the recent growth figures in Spain. Spain has been experiencing solid growth in the recent years: during the period 1999-2006, real annual GDP growth in Spain has averaged 3.2 percent, while it has averaged 2 percent in the EMU. In addition, the nontradable sector (services and construction) has been the main engine of growth, with an average growth rate of 3.5 percent, compared to a real growth rate in the tradable sector of 2.5 percent. Therefore, supply (productivity) factors cannot be the only explanation for the evolution of the inflation differential between Spain and the EMU, because declining productivity in the nontraded sector would imply higher inflation but lower output in this sector.[3] Therefore, to observe both an increase of output and prices in the nontradable sector, demand factors must have played an important role.

Finally, Angeloni and Ehrmann (2004), and Andrés et al. (2003) suggest that, due

[3]Output per employee is a rough measure of productivity, since it includes other factors that cannot be attributed to productivity shocks (composition effects of employment, for instance). However, other studies that have estimated total factor productivity (TFP) measures in both sectors (Gual et al., 2006) have found a similar pattern.

to different product and labor market structures, there is heterogeneity of inflation dynamics processes in each country of the union. As a result, even when economies are hit by symmetric shocks (such as oil prices, world demand, and the euro exchange rate), the response of inflation will be different across countries. Depending on the interaction between wage and price dynamics, second round effects could make inflation even more persistent.

All these hypotheses have been useful to explain the individual inflation country experiences of EMU member countries, and are not mutually exclusive. Surprisingly, the existing literature lacks a methodology to test their relative importance in explaning overall inflation differentals. This paper estimates a two-country, two-sector New Keynesian dynamic stochastic general equilibrium (DSGE) model of a currency union, using Spain and EMU data, and using Bayesian methods.[4] The main advantages of using a Bayesian approach are: first, information about the model's parameters can be introduced via the prior distribution. Second, from a computational point of view, it is helpful to identify the model's parameters (see Canova and Sala, 2006). This is particularly important in the present paper, because we use a relatively short sample that reflects the behavior of inflation and monetary policy under a currency area. Using a likelihood-based general equilibrium approach allows us to test all the implications of the model for explaining the data. Having specified a general equilibrium model with country and sector-specific demand and productivity shocks, and with (possibly) heterogeneous inflationary processes, we proceed to decompose the causes behind the inflation differentials between Spain and the EMU.

The results of the paper can be summarized as follows: first, the estimated degrees of nominal rigidity across countries and sectors are similar to those obtained with survey evidence, as summarized by Fabiani et al. (2006). Second, when the estimation is conducted allowing for effects of assuming positive steady-state inflation rates, these turn out to be unimportant for the structural parameter estimates, but increase the autocorrelation of the shocks significantly. Third, we cannot reject the hypothesis that the estimated coefficients that determine inflation dynamics in Spain and in the rest of the euro area are the same. Still, the impact of symmetric shocks can be different due to different composition of each country's CPI. Finally, the most important explanation for the inflation differential between Spain and the euro area

[4] See An and Schorfheide (2006) for a survey on the estimation of DSGE models using Bayesian methods.

comes from tradable sector productivity shocks that affect either Spain, the rest of the euro area, and both. On the other hand, nontradable technology shocks have a minor contribution to explain inflation differentials. Demand shocks are useful to explain a fraction of output growth volatility but not of inflation dispersion.

The rest of the paper is organized as follows: in section 2 we present a decomposition of the inflation differential based on the tradable and nontradable components of the HICP. In section 3 we outline the model, while section 4 briefly describes the Bayesian econometric approach. In Section 5 we present the results in terms of posterior parameter distributions, impulse responses, and second moments. Section 6 concludes.

2 Inflation Differentials between Spain and the EMU: What Drives Them?

From the policy perspective, the question to ask is to what extent are these inflation differentials (and the associated real exchange rate changes) important. Higher inflation in a country (or region) of a currency area reduces the purchasing power of its population, everything else equal. But the source of the inflation differential is also important: while higher nontradable inflation reduces real wages for domestic households, higher inflation in the tradable sector reduces competitiveness for the same type of good, with negative implications for output growth and employment. As we show in this section, the inflation differential between Spain and the euro area in the 2002-2006 period can be mostly explained by the behavior of the relative price of traded goods: this represents a loss of competitiveness of the Spanish economy vis-à-vis its trading partners, that could potentially damage the prospects of growth.

However, Spain has been growing faster than the EMU in the recent years, and hence real exchange rate and terms of trade (defined as the ratio of price of exports over price of imports) appreciation is indeed the expected mechanism through which adjustments would occur in a currency union. Large and persistent inflationary processes need not be "bad" per se, since countries growing above potential will have a tendency to have higher inflation, while countries in recession will tend to have lower inflation. As a result, countries in recession will experience a competitiveness gain, while those countries in the peak of their business cycle will suffer a loss:

altogether, the effect will be to bring all countries in a monetary union back to potential. Finally, it is worth noting that joining a monetary union can amplify economic fluctuations: the central bank reacts to average (EMU) inflation, but countries at the peak of their business cycle need higher rates than the union as a whole. Therefore, the real interest rate in a currency union is less countercyclical than under a country-specific inflation targeting regime, fluctuations become larger, and the mechanism that brings the union back to the steady state is by building up price differentials, as we have been observing in the recent years. The important issue is to ensure that structural rigidites in the economy do not imply a too large imbalance build-up due to inflation persistence, and hence that the adjustment occurs smoothly, rather than resulting in a painful recession.

We present the evolution of the price indices between Spain and its partners in the EMU, and decompose its evolution using a simple decomposition of the traded and nontraded components of the HICP, which we proxy by the "goods" and "services" components of the HICP, taken from Eurostat. The real exchange rate between Spain and the rest of the EMU can be expressed as

$$RER_t = \frac{P_t^*}{P_t}$$

where P_t^* is the price level of the rest of the EMU, and P_t is the price level in Spain. Figure 3 plots the evolution of the RER, after seasonally adjusting the series with the TRAMO/SEATS procedure.[5] The downward trend reflects the cumulative inflation differentials between Spain and the rest of the EMU since the launch of the euro in 1999.

To understand which components of the HICP are driving this behavior of inflation, we perform a simple decomposition of the real exchange rate (see Engel, 1999; Betts and Kehoe, 2006; and Chari et al. 2002). First, we multiply and divide the RER by the price of tradable goods in each country, such that we get:

$$RER_t = RER_t^T * RER_t^{REL}$$

where

$$RER_t^T = \frac{P_t^{T*}}{P_t^T}$$

[5] See Maravall (2002).

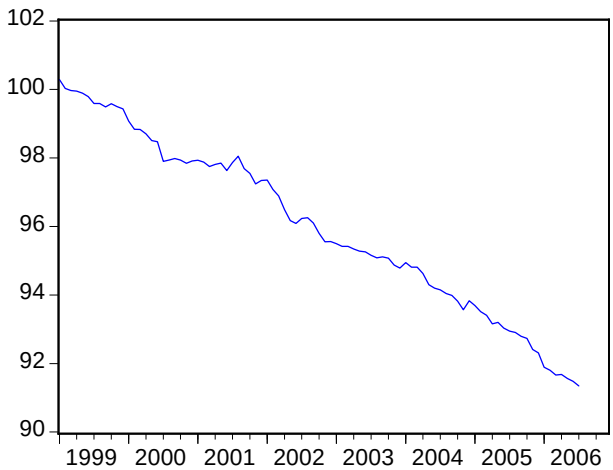

Figure 3: Real exchange rate between Spain and the rest of the EMU. Source: Eurostat and author's calculations.

and

$$RER_t^{REL} = \frac{\frac{P_t^T}{P_t}}{\frac{P_t^{T*}}{P_t^*}}.$$

Further, if we assume that in each country the CPI is a geometric average of traded and nontraded goods, then we have that $P_t = \left(P_t^T\right)^{\gamma}\left(P_t^N\right)^{1-\gamma}$, $P_t^* = \left(P_t^{T*}\right)^{\gamma^*}\left(P_t^{N*}\right)^{1-\gamma^*}$, then the expression for RER_t^{REL} becomes:

$$RER_t^{REL} = \frac{\left(\frac{P_t^T}{P_t^N}\right)^{1-\gamma}}{\left(\frac{P_t^{T*}}{P_t^{N*}}\right)^{1-\gamma^*}},$$

where γ and γ^* denote the fraction of traded goods in each country's HICP, and P_t^T, P_t^N, P_t^{T*}, P_t^{N*} are the prices of tradable (T) and nontradable (N) goods in both countries.

This procedure decomposes the evolution of the real exchange rate between the fluctuations of the price of traded goods in each country's CPI (RER_t^T), and the relative evolution of traded and nontraded goods prices in each country (RER_t^{REL}). The following expression holds for the change in the real exchange rate (lower case

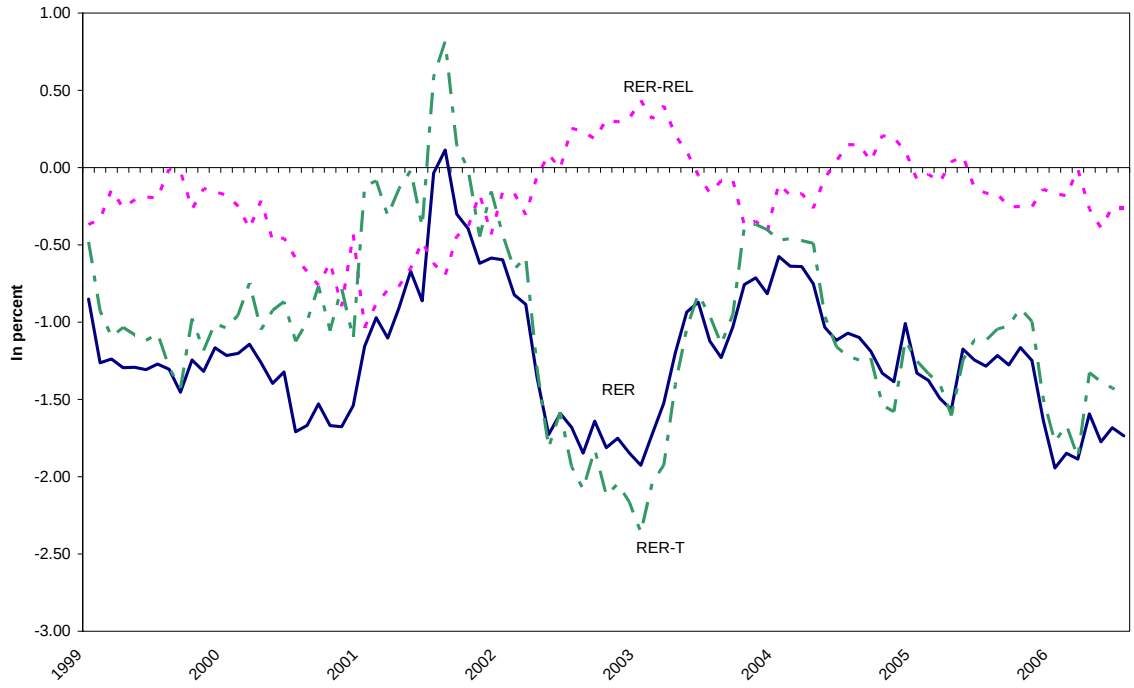

Figure 4: Decomposition of the inflation differential between tradable and nontradable inflation. Source: Eurostat and author's calculations.

variables denote logs, and Δ is the difference operator):

$$\begin{aligned} \Delta rer_t &= \Delta rer_t^T + \Delta rer_t^{REL} \\ &= \Delta p_t^{T^*} - \Delta p_t^T + \\ &\quad +(1-\gamma)(\Delta p_t^T - \Delta p_t^N) - (1-\gamma^*)(\Delta p_t^{T^*} - \Delta p_t^{N^*}) \end{aligned} \tag{1}$$

Therefore, deviations from purchasing power parity can be explained by: (i) deviations from the law of one price for tradable goods, and (ii) movements of relative prices between tradable and nontradable goods inside each country. If the fraction of tradable goods in the CPI is the same across countries $\gamma = \gamma^*$, and the law of one price holds for tradable goods, $\Delta p_t^{T^*} = \Delta p_t^T$, then fluctuations in the real exchange rate would be due to nontradable inflation only.[6] If either the consumption basket differs across countries, or there are deviations from the law of one price, or both, then fluctuations in the price of tradable goods will also matter. As we show in the following figure, this is indeed the case for Spain.

[6] This is the case analyzed by Altissimo et al. (2005).

Figure 4 presents this decomposition using annual rates (12-month changes). This evidence is purely data-based, and does not rely on a specific functional form for price indices (arithmetic or geometric weighted averages), since by construction, $RER_t^{REL} = RER_t/RER_t^T$. Clearly, there are two important subperiods since the launch of the euro that help explain inflation differentials (by definition, the evolution of the change in the real exchange rate in a currency union *is* the inflation differential). In the 1999-2001 period, both the relative price of goods across countries, as well as the movements of relative prices of goods and services inside each country, seemed to play a role in explaning the inflation differential. However, since 2002, virtually all the inflation differential can be explained by the evolution in the relative prices of tradable goods between Spain and the rest of the euro area. Table 2 confirms this analysis by presenting correlation coefficients between these three components, for the full sample 1999-2006 and for the two-subsamples. In all cases, the correlation between the aggregate inflation differential and its tradable component are always very close to one, and the correlation is highest in the 2002-2006 period, with a value of 0.92. On the contrary, the correlation between changes in the real exchange rate and the relative price component is mildly negative.[7] Finally, the correlation between the tradable and the relative price component is negative and high in absolute value.[8]

[7] This decomposition is done in terms of the overall real exchange rate, the traded goods real exchange rate, and the residual, and follows other papers in the literature. Another way to decompose the real exchange rate would have been to focus on the real exchange for nontraded goods ($RER^N = P^{N*}/P^N$), and a residual. In this case, the series RER and RER^N also display some strong comovement, but the evidence is not as strong as for the pair (RER, RER^T). For the full sample, the correlation between RER and RER^N is 0.47, while it increases to 0.66 for the 2002-2006 period.

[8] Using a similar decomposition, Engel (1999) and Chari et al. (2002) found that most of the variability in the real exchange rate between the United States and main trading partners was due to traded goods. On the other hand, Betts and Kehoe (2006), and Burnstein et al. (2005) suggest that the using the "goods only" component of the CPI is not a good measure of the prices of traded goods, because they include distribution, marketing, and other services that are of a nontraded nature. Using different proxies for the price of traded and nontraded goods, both papers show that the latter can explain up to 50 percent of the variability in the real exchange rate. Proxies used include the PPI for industrial goods, gross output deflators, and import and export price deflators at the dock.

Table 2: Correlation coefficients

	Full Sample	1999-2001	2002-2006
$\Delta rer_t, \Delta rer_t^T$	0.88	0.87	0.92
$\Delta rer_t, \Delta rer_t^{REL}$	-0.35	-0.10	-0.42
$\Delta rer_t^T, \Delta rer_t^{REL}$	-0.75	-0.57	-0.73

Source: Eurostat and author's calculations.

3 The Model

In order to model the interactions between Spain and the rest of the Euro Area inside a currency union, and based on the evidence presented in Section 2, we construct and estimate a two-country New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model of a currency union, similar to Duarte and Wolman (2002).[9] To study the behavior of inflation, the model introduces nominal rigidities. To test for the presence and importance of the Balassa-Samuelson effect, the model includes tradable and nontradable goods in both countries, and productivity shocks that affect all countries and sectors. Productivity shocks have the effect of improving the production frontier for each type of good, and hence cause an increase of output and a decrease of prices in that sector. In addition to country-specific productivity shocks, the model incorporates productivity shocks at the euro area level that affect either the tradable sector, or both sectors (to allow for technology spillovers across countries in the union).

To understand the role of demand factors, the model incorporates demand shocks in the form of government spending in both tradable and nontradable goods. These shocks will tend to move output and prices of a given sector in the same direction, and hence are able to explain a different comovement than productivity shocks. To understand the role of monetary factors, the model incorporates a monetary policy shock which is the residual of a Taylor-type interest rate rule that targets the EMU HICP. Finally, the model allows for the possibility that the inflation dynamics equations across countries and sectors are different, and a formal test can be conducted to contrast this hypothesis. Since the model features monopolistic competition and nominal rigidities, the price of tradable goods can differ across countries due to

[9] Other DSGE-based explanations of inflation differentials using models with traded and non-traded goods include Altissimo et al. (2005), and López-Salido et al. (2005).

productivity, demand, and monetary shocks. Therefore, following Altissimo et al. (2005), there is no price discrimination in the traded goods sector. To match persistence in real variables, habit formation in consumption is introduced, as in Smets and Wouters (2003).

3.1 Preferences

We assume that there are two countries in the european monetary union, home (H) and foreign (F), of unequal size. The home country is of size s, while the foreign country is of size $1-s$. Brands of traded goods are indexed by $h \in [0, s_1]$ in the domestic country and by $f \in [0, s_2]$ in the foreign country. Countries produce differentiated traded goods that are imperfect substitutes of each other. Brands of nontradable goods are indexed by $n \in [s_1, s]$ in the home country and by $n^* \in [s_2, 1-s]$.[10] We assume that technology and preferences is the same accross countries, but countries differ in the composition of the consumption indices, and in the degrees of nominal rigidity. The preferences of a household in the home country are assumed to be:[11]

$$U_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t - bC_{t-1}\right) - \frac{L_t^{1+\phi}}{1+\phi} \right] \right\}, \tag{2}$$

where E_0 denotes the expectation conditional on the information set at date $t=0$, and β is the intertemporal discount factor, with $0 < \beta < 1$. C_t denotes the level of consumption in period t, L_t denotes labor supply. The utility function displays external habit formation. $b \in [0,1]$ denotes the importance of the habit stock, which is last period's aggregate consumption. $\phi > 0$ is inverse elasticity of labor supply with respect to the real wage.

We define the consumption index as:

$$C_t \equiv \left[\gamma^{1/\varepsilon} \left(C_t^T\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{1/\varepsilon} \left(C_t^N\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where ε is elasticity of substitution between tradable (C_t^T) and non-tradable (C_t^N)

[10] The convention will be to use an asterisk to denote the counterpart in the foreign country of a variable in the home country (i.e. if aggregate consumption is C in the home country, it will be C^* in the foreign country and so on. The same applies to the model's parameters. When there is potential for confusion we try to explicitly clarifiy so.

[11] Rabanal and Tuesta (2006) study real exchange rate dynamics in a two country model, and allow for different preference parameters across countries. As we explain below, our sample period is quite short, and to limit the number of parameters to be estimated it is useful to impose some symmetry restrictions.

goods, and γ is the share of tradable goods in the consumption basket at home. The sub-index of consumption for traded goods is defined as:

$$C_t^T \equiv \left[\lambda^{\frac{1}{\theta}} \left(C_t^H\right)^{\frac{\theta-1}{\theta}} + (1-\lambda)^{\frac{1}{\theta}} \left(C_t^F\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

where θ is elasticity of substitution between home and foreign tradable goods, λ represents the degree of home bias in preferences. C_t^H and C_t^F are indexes of consumption across the continuum of differentiated goods produced in country H and F, and are given by:

$$C_t^H \equiv \left[\left(\frac{1}{s_1}\right)^{\frac{1}{\sigma}} \int_0^{s_1} c_t(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}}, C_t^F \equiv \left[\left(\frac{1}{s_2}\right)^{\frac{1}{\sigma}} \int_0^{s_2} c_t(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where $\sigma > 1$ is the elasticity of substitution across goods produced within country H, denoted by $c_t(h)$, and country F, denoted by $c_t(f)$. Note that C_t^H denotes consumption by home country nationals of domestically produced traded goods, while C_t^F denotes consumption by home country nationals of euro area traded goods. Similarly, the consumption of non-traded goods in the home country is given by

$$C_t^N \equiv \left[\left(\frac{1}{s_1 - s}\right)^{\frac{1}{\sigma}} \int_{s_1}^{s} c_t^N(n)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where $c_t^N(n)$ denotes the consumption of each individual non-traded good.

Individual demands for home and foreign tradables, and nontradable goods is given by:

$$\begin{aligned}
c_t(h) &= \frac{\lambda\gamma}{s_1}\left(\frac{p_t(h)}{P_t^H}\right)^{-\sigma}\left(\frac{P_t^H}{P_t^T}\right)^{-\theta}\left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t, \\
c_t(f) &= \frac{(1-\lambda)\gamma}{s_2}\left(\frac{p_t(f)}{P_t^F}\right)^{-\sigma}\left(\frac{P_t^F}{P_t^T}\right)^{-\theta}\left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t, \text{ and} \\
c_t^N(n) &= \frac{(1-\gamma)}{s - s_1}\left(\frac{p_t^N(n)}{P_t^N}\right)^{-\sigma}\left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t.
\end{aligned}$$

In this context, the home country consumer price index that corresponds to the previous specification is given by:

$$P_t \equiv \left[\gamma \left(P_t^T\right)^{1-\varepsilon} + (1-\gamma)\left(P_t^N\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{7}$$

where the home country price index for tradable goods has the following form:

$$P_t^T \equiv \left[\lambda \left(P_t^H\right)^{1-\theta} + (1-\lambda)\left(P_t^F\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{8}$$

with prices of home and foreign tradable goods, and non-tradable goods defined, respectively as:

$$P_t^H \equiv \left[\frac{1}{s_1}\int_0^{s_1} p_t(h)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}}, \; P_t^F \equiv \left[\frac{1}{s_2}\int_0^{s_2} p_t(f)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}}, \text{ and}$$

$$P_t^N \equiv \left[\frac{1}{s-s_1}\int_{s_1}^{s} p_t^N(n)^{1-\sigma}dn\right]^{\frac{1}{1-\sigma}},$$

where $p_t(i)$ for $i = h, f$, and $p_t^N(n)$ are prices sold in the home country, for both tradable and nontradable goods, respectively. Prices for the foreign country ($P_t^*, P_t^{H^*}$, $P_t^{F^*}$and $P_t^{N^*}$) are analogously defined, where γ^* is the fraction of tradable goods in the rest of the euro area consumption basket, and λ^* is the fraction of foreign-produced goods in the foreign consumption aggregate (i.e. the foreign degree of home bias).

For modelling simplicity, we assume that there are complete markets at the national and euro area-wide levels. In order to keep notation simple we do not explicitly introduce the portfolio of state-contingent assets that allows households to insure them against idiosyncratic risk, and that in equilibrium will be in zero net supply. We also assume that households have access to a riskless nominal bond denominated in euros (which, given the assumption of complete markets is redundant) that pays a gross rate of R_t. Then, the budget constraint of the domestic households in euros is given by:

$$\frac{B_t}{P_t R_t} \leq \frac{B_{t-1}}{P_t} + W_t L_t - C_t + \Pi_t \tag{9}$$

where W_t is the real wage, and Π_t are real profits for the home consumer.

The conditions characterizing the consumption/savings decisions is:

$$U_{C_t} = \beta E_t \left\{ U_{C_{t+1}} \frac{R_t P_t}{P_{t+1}} \right\} \tag{10}$$

where U_x is the partial derivative of the utility function with respect to variable x. Equation (10) corresponds to the consumption Euler equation of the home consumer. The first order conditions with respect to the labor supply implies the usual condition

that:

$$U_{L_t} = U_{C_t} W_t \tag{11}$$

where total labor is allocated between tradable and nontradable activities:

$$L_t = L_t^T + L_t^N$$

while combining optimality conditions between home and foreign households delivers the following condition for the real exchange rate, under complete markets:

$$RER_t = \frac{P_t^*}{P_t} = \nu \frac{U_{C^*}(C_t^*)}{U_C(C_t)} \tag{12}$$

3.2 The Government

In both countries, the government demands domestically produced traded and non-traded goods. The demand of the government has the same elasticties as the demand of the private sector:

$$\begin{aligned} g_t(h) &= \frac{\lambda\gamma}{s_1}\left(\frac{p_t(h)}{P_t^H}\right)^{-\sigma}\left(\frac{P_t^H}{P_t^T}\right)^{-\theta} G_t^T, \\ g_t(n) &= \frac{(1-\gamma)}{s-s_1}\left(\frac{p_t^N(n)}{P_t^N}\right)^{-\sigma} G_t^N. \end{aligned}$$

where G_t^T and G_t^N are exogenous process that follow AR(1) processes in logs.

3.3 Price Setting and Technology

In this model, suppliers behave as competitive monopolists when selling their products, subject to a Calvo-type restriction. In every period, intermediate goods producers receive a stochastic signal that allows them to change prices. This signal arrives with probability $1-\theta_N$ in the non-tradable sector, and $1-\theta_H$ in the tradable sector. In addition, we assume that a fraction φ_N in the nontradable sector, and φ_H in the tradable sector, index their price to the last period's inflation rate when they are not allowed to reoptimize. The model includes a euro-area technology shock with a unit root, that gives growth to the model. An advantage of this approach is that real variables in the model and in the data will be nonstationary in levels, but stationary in first differences, and hence provides a model-based method

to detrend the data. The model also includes stationary technology shocks in the traded and nontraded sectors for each country, and we assume that the innovations to the traded sector technology shock are correlated across countries.

3.3.1 Non-Tradable Sector

Each firm produces according to the following production function

$$y_t^N(n) = Z_t^N L_t^N X_t \tag{13}$$

where X_t is a labor augmenting aggregate euro-area wide technology shock which has a unit root, as in Galí and Rabanal (2005), and Rabanal and Tuesta (2006):

$$\log X_t = x + \log X_{t-1} + \varepsilon_t^x \tag{14}$$

Hence, along the balanced growth path, real variables in both countries grow at a rate x. Z_t^N is the country-specific productivity shock to the non-tradable sector at time t which evolves according to an AR(1) process in logs

$$\log Z_t^N = \log(\bar{Z}^N) + \rho^{Z,N} \log Z_{t-1}^N + \varepsilon_t^{Z,N} \tag{15}$$

Firms in the non-tradable sector face the following maximization problem:

$$Max_{p_t^N(n)} E_t \sum_{k=0}^{\infty} \theta_N^k \Lambda_{t,t+k} \left\{ \left[\frac{p_t^N(n) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N} - MC_{t+k}^N}{P_{t+k}^N} \right] y_{t+k}^{N,d}(n) \right\} \tag{16}$$

subject to

$$y_{t+k}^{N,d}(n) = \frac{(1-\gamma)}{s-s_1} \left(\frac{p_t^N(n)}{P_{t+k}^N} \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N} \right)^{-\sigma} Y_{t+k}^N \tag{17}$$

where $\Lambda_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor, with $\lambda_t = U_C(C_t)$, $y_t^{N,d}(n)$ is total individual demand for a given type of nontraded good, and Y_t^N is aggregate demand for nontraded goods, consisting of private consumption and government spending:

$$Y_t^N = C_t^N + G_t^N$$

MC_t^N corresponds to the nominal marginal cost in the non-tradable sector. From cost minimization:

$$MC_t^N = P_t \frac{W_t}{Z_t^N X_t}$$

The supplier maximizes (16) with respect to $p_t^N(n)$ given the demand function (17) and taking as given the sequences of all other prices. The optimal choice in the symmetric equilibrium is given by:

$$\frac{\hat{p}_t^N}{P_t^N} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \theta_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N} \right)^{-\sigma} \frac{MC_{t+k}^N}{P_{t+k}^N} Y_{t+k}^N}{\sum_{k=0}^{\infty} \beta^k \theta_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N} \right)^{1-\sigma} Y_{t+k}^N} \right\}$$

To analyze the effects of non-zero inflation rates, it will be convenient to express the price-setting equations in a recursive way. We write them as follows:

$$\frac{\hat{p}_t^N}{P_t^N} K_t^{N,1} = \frac{\sigma}{(\sigma-1)} K_t^{N,2}$$

where

$$\begin{aligned} K_t^{N,1} &= E_t \sum_{k=0}^{\infty} \beta^k \theta_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N} \right)^{1-\sigma} Y_{t+k}^N \\ &= \lambda_t Y_t^N + \beta \theta_N E_t \left\{ \left[\frac{\left(\Pi_t^N\right)^{\varphi_N}}{\Pi_{t+1}^N} \right]^{1-\sigma} K_{t+1}^{N,1} \right\}, \end{aligned}$$

and

$$\begin{aligned} K_t^{N,2} &= \sum_{k=0}^{\infty} \beta^k \theta_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N} \right)^{-\sigma} \frac{MC_{t+k}^N}{P_{t+k}^N} Y_{t+k}^N \\ &= \lambda_t \frac{MC_t^N}{P_t^N} Y_t^N + \beta \theta_N E_t \left\{ \left[\frac{\left(\Pi_t^N\right)^{\varphi_N}}{\Pi_{t+1}^N} \right]^{-\sigma} K_{t+1}^{N,2} \right\} \end{aligned}$$

The evolution of the price level of non-tradables is

$$P_t^N \equiv \left[\theta_N \left(P_{t-1}^N \left(\Pi_{t-1}^N\right)^{\varphi_N} \right)^{1-\sigma} + (1-\theta_N) \left(\hat{p}_t^N\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$.

3.3.2 Tradable Sector

Most expressions in the tradable sector are analogous to those of the nontradable sector. Each firm produces according to the following production function

$$y_t^H(h) = Z_t^T L_t^T X_t \tag{18}$$

Z_t^T is the country-specific productivity shock to the tradable sector at time t which evolves:

$$\log Z_t^T = \log(\bar{Z}^T) + \rho^{Z,T} \log Z_{t-1}^T + \varepsilon_t^{Z,T} + \varepsilon_t^Z \tag{19}$$

Note that the innovation of the tradable shock has a country-specific component, $\varepsilon_t^{Z,T}$, and a euro-area component, ε_t^Z. As long as the standard deviation of ε_t^Z is positive, there will be some correlation in the tradable sector productivity shocks, as in most of the International Real Business Cycle literature (see Stockman and Tesar, 1995).

Firms cannot price-discriminate in the currency area, and set the price in euros to sell in both markets, facing a downward sloping demand. Proceeding the same way as with the nontradable sector, we arrive at the following optimal expressions:

$$\frac{\hat{p}_t^H}{P_t^H} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \theta_H^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^H\right)^{\varphi_H}}{\Pi_{t+s}^H} \right)^{-\sigma} \frac{MC_{t+k}^H}{P_{t+k}^H} Y_{t+k}^H}{\sum_{k=0}^{\infty} \beta^k \theta_H^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^H\right)^{\varphi_H}}{\Pi_{t+s}^H} \right)^{1-\sigma} Y_{t+k}^H} \right\} \tag{20}$$

where

$$MC_t^H = P_t \frac{W_t}{Z_t^H X_t}$$

The evolution of the price level of tradables is

$$P_t^H \equiv \left[\theta_H \left(P_{t-1}^H \left(\Pi_{t-1}^H \right)^{\varphi_H} \right)^{1-\sigma} + (1-\theta_H) \left(\hat{p}_t^H \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^H = \frac{P_{t-1}^H}{P_{t-2}^H}$.

3.4 Monetary Policy

In order to abstract from fiscal policy considerations, it is assumed that government spending in the two areas is financed through lump sum taxes. Monetary policy is conducted by the ECB with a Taylor rule that only targets the EMU HICP:

$$R_t = \bar{R}^{1-\rho_r} R_{t-1}^{\rho_r} \left(\frac{\Pi_t^{EMU}}{\bar{\Pi}^{EMU}}\right)^{(1-\rho_r)\gamma_\pi} \exp(\varepsilon_t^m) \tag{21}$$

where ε_t^m is an iid monetary policy shock. The European HICP is defined as

$$P_t^{EMU} = P_t^s \left(P_t^*\right)^{1-s}$$

where s is the size of the home economy.

3.5 Market Clearing

The market clearing condition in the tradable goods sector at home is

$$y_t^H(h) = c_t(h) + c_t^*(h) + g_t(h), \text{ for } \forall h \in [0, s_1] \tag{22}$$

The market clearing for the nontradable goods sector is

$$y_t^N(n) = c_t(n) + g_t(n), \text{ for } \forall n \in [s_1, s] \tag{23}$$

In the aggregate the following conditions hold:

$$Y_t^H = C_t^H + C_t^{H^*} + G_t^T$$

$$Y_t^N = C_t^N + G_t^N$$

Aggregate real GDP aggregates traded and nontraded goods using the appropiate relative prices:

$$Y_t = \frac{P_t^H}{P_t} Y_t^H + \frac{P_t^N}{P_t} Y_t^N \tag{24}$$

3.6 Inflation Dynamics and The Effects of Steady State Inflation

The model's dynamics are obtained by taking a linear approximation around the steady state with positive rates of real growth and inflation. As in King, Plosser and Rebelo (1988), real variables grow at the same rate as the EMU-wide technology shock. Hence, consumption, output, real wages, and government spending in each country are normalized by the level of technology.

In this paper we take a novel approach with respect to the existing literature on estimating DSGE models and we consider the effects of a positive rate of inflation in the steady state on inflation dynamics. It is customary in these models to demean the inflation rate and assume that in the steady state, the gross rate of inflation is zero.[12] However, we show below that all steady-state relationships depend on the steady-state state of inflation. This is typically ignored in the empirical literature (but not in the theoretical one, see Schmitt-Grohé and Uribe, 2006). Because of trend inflation, optimal price setters will not set the same price as those who do not in the steady state, unless there is full indexation to the steady-state inflation rate. Appendix A discusses how marginal costs can differ across sectors and countries due to heterogeneous Phillips Curve parameters and non-zero steady-state inflation. It discusses how selecting the appropiate values for the levels of productivity in each country/sector will ensure the same prices in steady state.

Taking a loglinear approximation of the pricing equations for the nontraded sector at home delivers:

$$(\Pi_t^*)^N + k_t^{N,1} = k_t^{N,2} \tag{25}$$

where

$$k_t^{N,1} = [1-\beta\theta_N\Pi^{-(1-\varphi_N)(1-\sigma)}](\tilde{\lambda}_t+\tilde{y}_t^N)+\beta\theta_N\Pi^{-(1-\varphi_N)(1-\sigma)}[k_{t+1}^{N,1}-(1-\sigma)(\Delta p_{t+1}^N-\varphi_N\Delta p_t^N)] \tag{26}$$

$$k_t^{N,2} = [1-\beta\theta_N\Pi^{(1-\varphi_N)\sigma)}](\tilde{\lambda}_t+\tilde{y}_t^N+mc_t^N)+\beta\theta_N\Pi^{(1-\varphi_N)\sigma}[k_{t+1}^{N,2}+\sigma(\Delta p_{t+1}^N-\varphi_N\Delta p_t^N)] \tag{27}$$

[12] See Smets and Wouters (2003), and Rabanal and Rubio-Ramírez (2005) among many others.

and $(\Pi_t^*)^N = \frac{\hat{p}_t^N}{P_t^N}$. The evolution of the inflation rate of non-tradables is

$$(\Pi_t^*)^N = \frac{\theta_N \Pi^{-(1-\varphi_N)(1-\sigma)}}{(1-\theta_N)((\Pi^*)^N)^{1-\sigma}}(\Delta p_t^N - \varphi_N \Delta p_{t-1}^N) \tag{28}$$

where

$$(\Pi^*)^N = \left[\frac{1-\theta_N\left(\Pi^N\right)^{-(1-\varphi_N)(1-\sigma)}}{(1-\theta_N)}\right]^{\frac{1}{1-\sigma}}$$

and $mc_t^N = \log(\frac{MC_t^N}{P_t}) - \log(\frac{MC^N}{P})$.

If steady state inflation is zero ($\Pi^N = \Pi = 1$), then the previous four equations boil down to the familiar New Keynesian Phillips Curve with backward looking behavior:

$$\Delta p_t^N - \varphi_N \Delta p_{t-1}^N = \beta E_t(\Delta p_{t+1}^N - \varphi_N \Delta p_t^N) + \kappa^N mc_t^N \tag{29}$$

where $\kappa^N = \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N}$. In the empirical part, we consider the two cases, one in which all inflation rates are demeaned, and we assume that $\Pi = 1$, and the other case where we allow for positive trend inflation, and also estimate the target for the European Central Bank.

4 Parameter Estimation

The goal of this paper is to estimate the parameters of the two-country, two-sector model using data for Spain and the euro area, and using Bayesian methods. In this section we briefly sketch how to make the Bayesian estimation method operational. Denote by $\{\chi_t\}_{t=1}^T$ the set of observable variables that we wish to explain, and Θ the vector of parameters of the model (including preferences, technology, government policies, and stochastic properties of the shocks). From Bayes rule, the posterior distribution of the model's parameters is proportional to the product of the likelihood function $\mathfrak{L}\left(\{\chi_t\}_{t=1}^T|\Theta\right)$ and the prior distribution $\Pi(\Theta)$:

$$P(\Theta|\{\chi_t\}_{t=1}^T) \propto \Pi(\Theta)\mathfrak{L}\left(\{\chi_t\}_{t=1}^T|\Theta\right)$$

Prior information about the model's parameters is introduced in the $\Pi(.)$ function. In order to evaluate the likelihood function, denote by S_t the set of all endogenous variables of the model (either state or forward looking, observable or not, expressed

in log-deviations from steady-state values), and by Ψ_t the set of all shocks. The linearized system with rational expectations is solved with standard methods, and the law of motion of the model can be written as:[13]

$$\begin{aligned} S_t &= A(\Theta)S_{t-1} + B(\Theta)\Psi_t \\ \Psi_t &= C(\Theta)\Psi_{t-1} + D(\Theta)\varepsilon_t \end{aligned} \tag{30}$$

where $E(\varepsilon_t\varepsilon_t') = I$, ε_t is the vector of structural innovations, and the matrices A, B, C, D are functions of the parameters of the model, Θ. We complement the law of motion of the model with a measurement equation:

$$\chi_t = HS_t \tag{31}$$

where the H has zeros everywhere, and a one in each row to select a subset of observable variables from S_t. The system of equations (30)-(31) is the usual state-space representation of a model, with the first set of equations being the *transition* equation, and the second equation being the *measurement* equation. Then, for a given vector of parameters Θ, the likelihood function can be evaluated applying standard Kalman filter formulas.

The prior might involve non-normal distributions, and the likelihood function does not have a closed-form expression. As a result, it is not possible to obtain an analytical expression for the posterior. Since we are able to numerically evaluate the prior and the likelihood function, then it is possible to numerically construct the posterior distribution of the model's parameters by making use of Markov Chain Monte Carlo (MCMC) methods.[14] Basically, they amount to efficiently draw from the posterior distribution, and obtain a time series of Θ, from which we can compute the posterior moments of the model's parameters, as well as posterior impulse responses and second moments. In our case, we make use of the Metropolis-Hastings algorithm and obtain 125,000 draws, after allowing for a burn-in phase of 25,000 draws.

[13] In Appendix B, we present the full set of loglinearized equations of the model.
[14] See An and Schorfheide (2006) for a description.

4.1 Data

4.1.1 Choice of Sample Period

We face severe data restrictions when estimating the model using Euro Area data. The euro and the common monetary policy were launched in January 1st, 1999, and this paper attempts to study the behavior of inflation in a currency union. At a quarterly frequency, the sample consists of 30 observations, which represent too few observations, given that the model has a fair amount of parameters. For asymptotic reasons, it is desirable to have the longest possible time series, and several papers have used the Area Wide Model (AWM) data set of Fagan, Henry and Mestre (2001) to estimate models of the Euro Area as a whole.[15] By making this choice, one implictly assumes that the Euro Area behaved like a common currency area since the beginning of the sample period (the 1970s, or the 1980s). This can be a difficult assumption to accept, specially for those countries who joined the European Union (EU) and the European Monetary System (EMS) over the years. The assumption of a common monetary policy might be a good approximation for the countries in the "core" of the old EMS, whose monetary policies closely followed the Bundesbank in the 1980s and 1990s. For instance, Pytlarczyk (2005), estimates a model of Germany inside the euro area. He does so by estimating two models at the same time: a model of a currency area like the one presented here from 1999 onwards, and a model of fixed exchange rates before the launch of the euro. In the second case he introduces risk premia to model interest rate differentials in a fixed exchange rate regime.

Spain joined the EU in 1986, and the EMS in 1989, and it launched inflation targeting in 1995 to converge in nominal terms with the rest of countries of the euro area. Therefore, it is difficult to accept the assumption that Spain belonged to some european entity that behaved as a "synthetic" currency union, and hence this paper does not follow Pytlarczyk's (2005) approach. The structural change for Spain of joining the EMU was a larger structural break than for Germany. Figure 5 presents the 3-month T-bill rate in Spain, Germany, an average of the euro area before 1999, and the euro area 3-month T-bill after 1999. Monetary policy in Spain did not follow that of the Bundesbank or a european aggregate during the 1980s and even most of the 1990s. Convergence in interest rates only seemed to happen after 1997: as Figure 6 shows, the spread between Spain's 3-month rates and the average of the

[15] This is a synthetic, nonofficial dataset maintained at the Econometric Modelling Unit of the ECB. For two examples, see Smets and Wouters (2003), and Rabanal and Tuesta (2006).

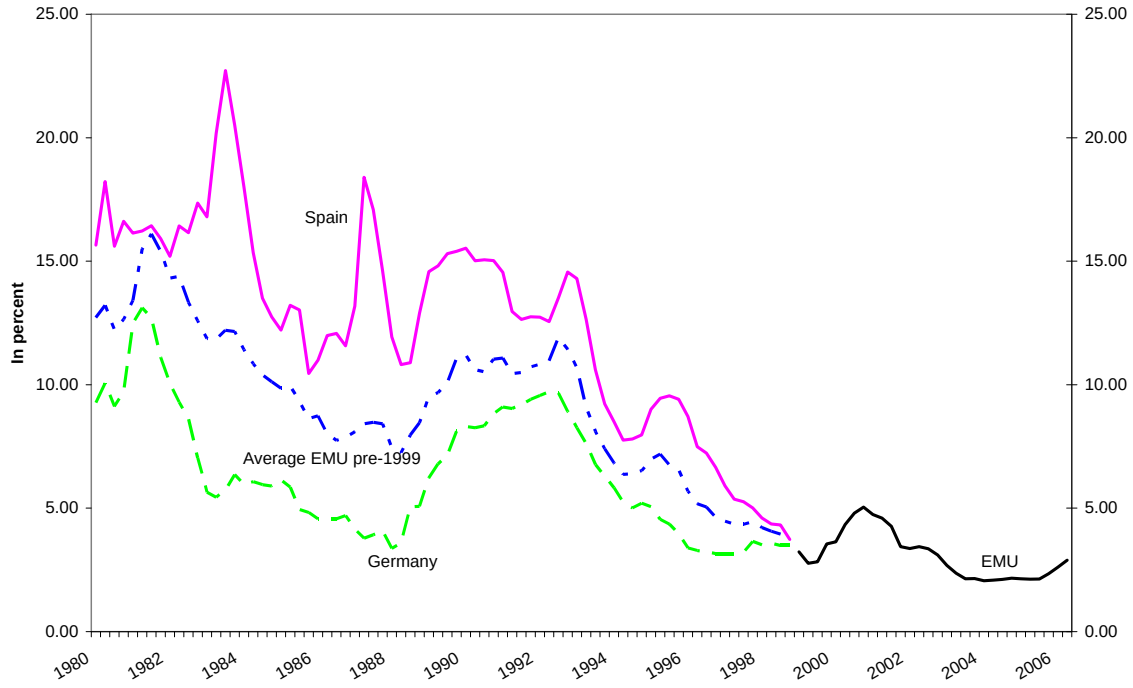

Figure 5: Interest Rates, 3 Month Treasury Bills. Source: Eurostat and ECB.

Euro area became less than 50 basis points in the last 20 years only after the fourth quarter of 1997. Afterwards, the spread kept declining to insignificant levels, once it became clear during 1998 that Spain would enter in the EMU.

Finally, Figure 7 shows the 12-month CPI inflation rate. For the whole sample period, and specially in the 1980s and early 1990s Spain experienced higher inflation than the euro area. Focusing on more recent periods, average inflation in the euro area countries crossed the 4 percent threshold in 1992:03, and has stayed below that value ever since. In Spain, it took three and a half more years for inflation to fall under 4 percent (in 1996:01), after more than two decades of higher inflation rates.

For all the reasons we have explained in this subsection, and to address the fact that there was a structural change in Spain in the process of converging in nominal terms to the Euro Area, we start our sample period in 1996:01. This leaves our sample with 42 observations. Clearly, this is a short sample, and only with time we will be able to estimate this model with more observations from the EMU period.

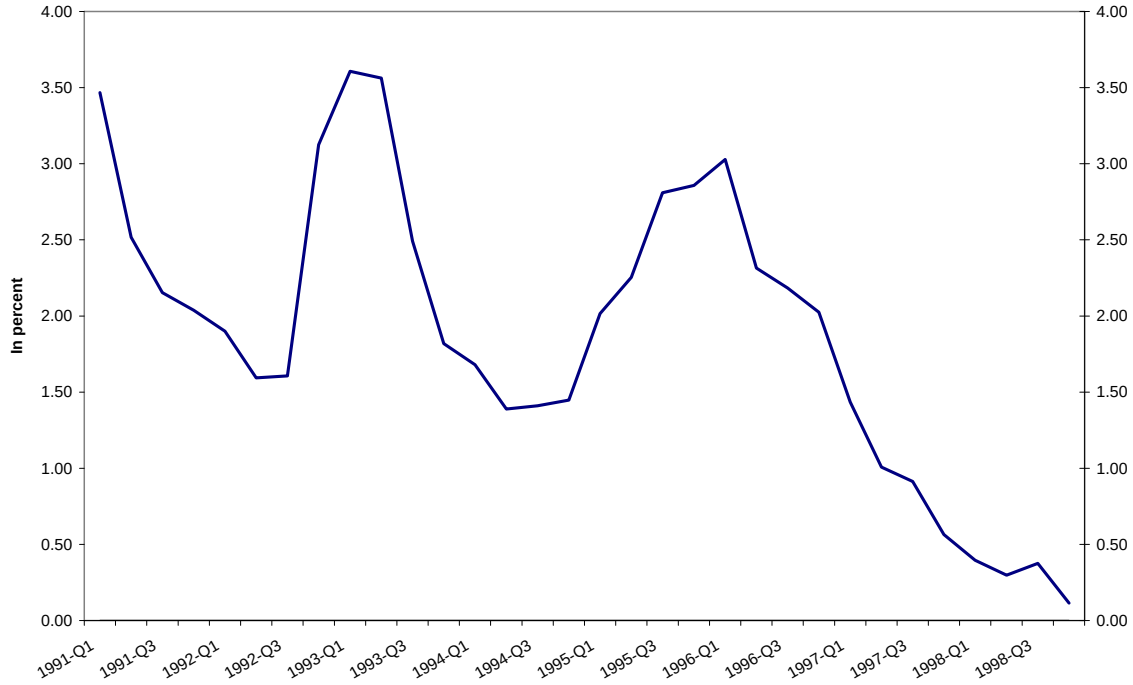

Figure 6: Spread between Spain and average EMU 3 month rates. Source: Eurostat and ECB.

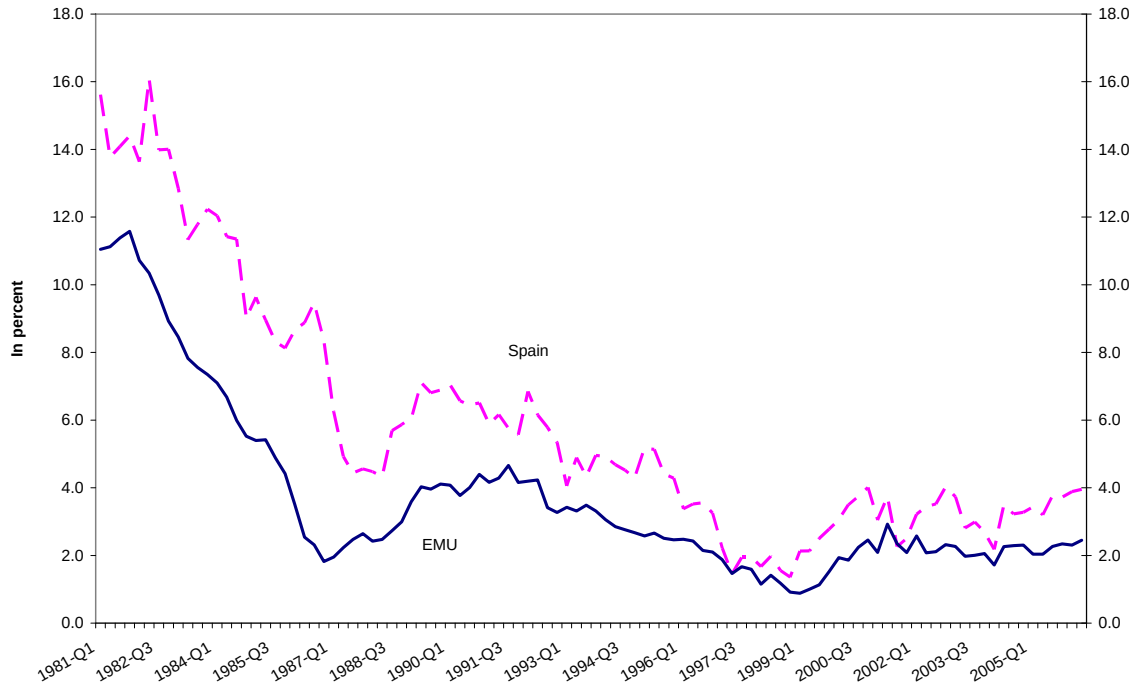

Figure 7: CPI Inflation Rates. Source: Eurostat.

4.1.2 Variables

The model presented in Section 3 is estimated with eight variables. These variables are quarterly HICP inflation, quarterly HICP Services inflation, quarterly real GDP growth rates for both Spain and EMU, quarterly nontradable (Services and Construction) real GDP growth for Spain, and the 3-month T-bill rate (between 1996-1998, we use an average of the euro area's 3-month t-bill).[16] We take logs and first differences of the real GDP and price level series to obtain quarterly growth rates. We divide the interest rate by four to obtain a quarterly equivalent. The home country is Spain and the foreign country is the rest of the euro area.

We estimate the model in two different ways: in the first one, we demean all variables and assume that growth and inflation are zero in the steady state. This is not the model-consistent way of detrending the data, but has become standard practice in the literature that estimates DSGE models with Bayesian methods. We refer to the data set where all variables are demeaned as $\{\chi_{1,t}\}_{t=1}^{T}$ and the model assumes that $x = \log(\Pi) = 0$.

In the second case, we enter all variables without demeaning and estimate the constants in a steady state-consistent way. In this case, the likelihood is evaluated for vector of observed variables that we denote by $\mathfrak{L}\left(\left\{\chi_{2,t} - c\right\}_{t=1}^{T} |\Theta\right)$ where $\chi_{2,t} = \{\Delta p_t,\ \Delta p_t^{EMU},\ \Delta p_t^{N},\ \Delta p_t^{N,EMU},\ \Delta y_t,\ \Delta y_t^{EMU},\ \Delta y_t^{N},\ r_t\}$, and $c = \{\log(\Pi),\ \log(\Pi),\ \log(\Pi),\ \log(\Pi),\ x,\ x,\ x,\ \log[\frac{(1+x)\Pi}{\beta}]\}$. In addition, the vector of parameters describing the model includes x and Π, and as we have shown in section 3.6, steady-state inflation affects the model's dynamics. This is a clearly a restrictive assumption, since we know that Spain has been experiencing higher inflation and growth than the EMU as a whole, and that services inflation is higher than goods inflation. However, it is the model-consistent way, and will help us understand if the steady-state effects of a positive steady-state inflation rate are important or not.

4.2 Priors

Since our sample is short, we opt for calibrating most of the parameters of the model (Θ), and focus on estimating the coefficients of the Taylor rule, the degrees

[16] All EMU data come from Eurostat, as well as Spain's HICP and its services components. Spain's sectoral GDP data come from the Instituto Nacional de Estadística (INE, National Statistics Institute).

of nominal rigidity in each sector and country, and the autoregressive parameters and standard deviations of the shocks. In order to further reduce the number of parameters to be estimated, we assume that the AR coefficients of the shocks are the same for each type of shock. Table 2 presents the parameter values that we calibrate, and the sources that we use.

The steady state nominal interest rate is $\log\left[\frac{(1+x)\Pi}{\beta}\right]$. As Ireland (2004) points out, in models where technology follows a unit root with drift, it is difficult to find reasonable calibrations for the three parameters (x, Π, β) that deliver at the same time a reasonable steady-state value for the nominal interest rate. For instance, if we assume that both the real growth rate and the inflation rate are 2 per cent annual rate, this is enough to make the interest rate 4 percent annual rate in the steady state, which is slightly above what we observe in the data for the recent years. Hence, we need calibrations of β very close to one. Lower calibrated values for β will imply that when we estimate the model with the constant terms, we are likely to obtain unrealistically low estimates for x or Π, and we will not be able to fit the nominal interest and the inflation rate and the growth rate at the same time. To keep the problem of the consumer bounded, we calibrate β to 0.999.

Table 2: Calibrated Parameters

Parameter	Value	Source
β	0.999	
ϕ, ϕ^*	1	Rabanal and Rubio-Ramirez (2005)
b, b^*	0.6	Rabanal and Tuesta (2006)
ε	0.44	Stockman and Tesar (1995)
θ	1	Galí and Monacelli (2006)
σ	10	Duarte and Wolman (2002)
s	0.1	Average weight of Spain HCPI in EMU HCPI since 1996
η	0.18	Average ratio G/Y in Spain since 1996
η^*	0.20	Average ratio G/Y in Euro Area since 1996
γ	.66	Proportion of goods in Spain HCPI
γ^*	.61	Proportion of goods in Euro Area HCPI
λ	.16	Average ratio of imports from EMU over total spending
λ^*	.015	Average ratio of imports from Spain over total spending

For the parameters involving bilateral trade (λ, λ^*), government spending (η, η^*) or the size of sectors (γ, γ^*), we use national accounts data and Eurostat. For the size of the Spanish economy inside the EMU HICP (s), we use the average weight for the sample period (weights are revised every year). Then, we calibrate the parameters denoting habit formation (b, b^*), the inverse elasticity of labor supply (ϕ, ϕ^*), and the various trade elasticities $(\varepsilon, \theta, \sigma)$, using studies that have estimated models for the euro area, or other research papers where calibration is used. We are aware that the choice of the calibrated parameters affects the estimated ones. As more data become available, we would like to be able to estimate these as well. When we estimate the model with non-zero inflation, we adjust the ratio of productivity levels $(\Phi = \frac{Z^T}{Z^N})$ such that the price levels are the same across sectors and countries in the steady state.[17]

Table 3: Priors

Parameter	Distribution	Mean	Std. Dev.
$\theta_N, \theta_H, \theta_N, \theta_{F^*}$	*Beta*	0.75	0.15
$\varphi_N, \varphi_H, \varphi_N, \varphi_{F^*}$	*Beta*	0.6	0.2
x	*Normal*	0.006	0.001
Π	*Normal*	1.005	0.001
γ_Π	*Normal*	1.5	0.1
ρ_R	*Beta*	0.7	0.01
$\rho^{Z,N}, \rho^{Z,T}, \rho^{G,N}, \rho^{G,T},$	*Beta*	0.7	0.01
$\sigma(\varepsilon_t^{X,i}), i = N, T, N^*, T^*$	*Gamma*	0.7	0.3
$\sigma(\varepsilon_t^{G,i}), i = N, T, N^*, T^*$	*Gamma*	1	0.5
$\sigma(\varepsilon_t^m)$	*Gamma*	0.4	0.2

Table 3 displays the prior distributions over the estimated parameters. We assume that all Calvo lotteries have a prior mean probability of 0.75, implying that prices are reset optimally every 4 quarters. These values are in line with the survey evidence in Fabiani et al. (2006). The degree of indexation has a prior mean of 0.6, which is somewhat larger than the survey evidence presented in Fabiani et al. (2006), but tries to reflect the fact that inflation differentials are highly persistent. At any rate, the standard deviation is large enough to accomodate a wide enough range

[17] See Appendix A for details.

of parameter values. When we estimate the model with constants, we assume that the growth rate is 0.6 percent quarterly, which is in between the Euro Area and Spain's average growth rates. The steady-state rate of inflation has a prior mean of 0.5 percent quarterly, which would be consistent with the ECB's stated inflation objective of 2 percent annual inflation (or below). The Taylor rule coefficients have prior means which are quite conventional in the literature, the reaction to inflation respects the Taylor principle, and we restrict the parameters of the model to the region where it has a unique, stable solution. The prior distribution over the productivity and demand shocks autorregresive coefficients have prior means of 0.7 and large enough standard deviations to allow for other values. To reduce the parameter space, we have assumed that the AR coefficients are the same for the same type of shock across countries (i.e $\rho^{i,j} = \rho^{i,j^*}$, for $i = Z, G$, and $j = T, N$). Different volatilities of the same type of shock across countries are allowed through different standard deviations of the innovations, which have Gamma prior distributions, to ensure positive numbers.

5 Results

5.1 Posterior

Table 4 presents the posterior mean and 90 percent confidence interval for the model's parameters. We present the posteriors for the relevant model parameters, and leave the posterior distribution of the shocks coefficients to Appendix C. The estimates for the Calvo lottery parameters are smaller than the prior mean for the tradable sector and larger for the nontradable sector. In the case of tradable goods, the posterior estimate for the Spanish sector is 0.49, while it turns out to be 0.51 for the rest of the euro area. The implication is that average price durations are about 2 quarters both in Spain and the rest of the euro area. However, the posterior mean for the Calvo lottery in the nontradable sector is 0.77 in Spain and 0.86 in the euro area, implying that posterior average durations range between 4 quarters in Spain and 6 quarters in the est of the euro area. The degrees of backward looking behavior in the Phillips Curve amount to being about one half for the Spanish tradable sector case, and roughly two-fifths for the rest of the euro area. For the non-tradable sector, while the estimates point to a higher degree of nominal stickiness, the degrees of backward looking behavior are smaller, in the range of one-fourth in Spain and

less than ten percent in the euro area. All these results are fully consistent with the survey evidence presented by Fabiani et al. (2006).

Table 4. Posterior Distributions

	Heterogeneous		*Homogeneous*	
	Raw Data	*Demeaned*	*Raw Data*	*Demeaned*
θ_H	0.49 (0.39- 0.61)	0.47 (0.35- 0.58)	0.52 (0.42- 0.62)	0.49 (0.39- 0.59)
θ_{F*}	0.51 (0.39- 0.62)	0.49 (0.37- 0.61)	–	–
θ_N	0.77 (0.71 - 0.81)	0.69 (0.63 - 0.76)	0.83 (0.81 - 0.86)	0.77 (0.73 - 0.80)
θ_{N*}	0.86 (0.83 - 0.90)	0.81 (0.77 - 0.85)	–	–
φ_H	0.50 (0.17 - 0.85)	0.40 (0.09 - 0.71)	0.31 (0.05 - 0.58)	0.30 (0.04 - 0.57)
φ_{F*}	0.39 (0.08 - 0.69)	0.39 (0.08 - 0.71)	–	–
φ_N	0.25 (0.04 - 0.47)	0.19 (0.03 - 0.36)	0.07 (0.01 - 0.13)	0.08 (0.01 - 0.15)
φ_{N*}	0.08 (0.01 - 0.14)	0.15 (0.02 - 0.26)	–	–
γ_Π	1.49 (1.32 - 1.69)	1.54 (1.38 - 1.69)	1.51 (1.35 - 1.66)	1.54 (1.38 - 1.69)
ρ_R	0.65 (0.57 - 0.74)	0.47 (0.36 - 0.57)	0.64 (0.56 - 0.73)	0.46 (0.36 - 0.56)
Π	1.0053 (1.0042 - 1.0063)	1	1.0051 (1.0042 - 1.0061)	1
x	0.0048 (0.004 - 0.0057)	0	0.0052 (0.004 - 0.006)	0
Log L	-169.69	-52.58	-160.51	-47.92

The estimates for the Taylor rule suggest that the ECB targets inflation with a large coefficient on the reaction of nominal interest rates to inflation, of about 1.5, with a significant degree of nominal inerta, of 0.65. Note that these estimates are not so different from the priors, such that given these, the information contained in the likelihood function does not provide additional information. The estimate for the steady state inflation rate is right above 2 percent at an annual rate, while for the real growth rate of the economy it is right below 2 percent. This estimate is on the low side and reflects the tension we mentioned previously on the fact that this parameter tries to fit the real GDP growth rates and the nominal interest rate at the same time.

The second column displays the parameter estimates of the model where all variables

have zero mean, and we assume that the growth rates of prices and real activity in the balanced growth path are zero. None of the parameter estimates change much, specially those of the Phillips Curves. The only noticeable difference is that the posterior mean of the interest rate smoothing parameter goes from 0.65 to 0.47. There does not seem to be too much action from assuming positive steady-state inflation rates on the estimated structural coefficients of the model. However, the posterior estimates for the AR coefficients of the shocks decrease significantly (see Appendix C). In the case with raw data, the AR coefficients are in the range between 0.83 for the tradable sector technology shock to 0.98 for the nontradable sector technology shock. On the contrary, the AR coefficients in the case of demeaned data are in the range of 0.67 to 0.77. Clearly, in the model with raw data and where sample inflation and growth means are assumed to be equal across sectors and countries, higher autocorrelated shocks are needed to keep variables away from their theoretical means. Finally, note that under demeaned data the marginal likelihood improves significantly from -169.69 to -52.58, denoting that the restriction of common growth rates and inflation rates across countries and sectors is rejected in the data.

In the third and fourth columns we show the parameter estimates when we impose that the coefficients of the Phillips Curves are the same across countries (that is, $\theta_H = \theta_{F^*}$, $\theta_N = \theta_{N^*}$, $\varphi_H = \varphi_{F^*}$, $\varphi_N = \varphi_{N^*}$). In this case, price stickiness in the tradable sector is about 2 quarters between optimal price changes, while in the nontradable sector, prices change optimally every 6 quarters). On the other hand, there is more backward looking behavior in the tradable sector inflation, with about one-third of firms following a backward looking price indexation rule. In the nontradable sector, backward looking behavior is less important quantitatively. The results for the model without steady-state inflation and demeaned variables only provide marginal changes. Overall, the marginal likelihoods increase in both cases, thereby rejecting the hypothesis put forth by Angeloni and Ehrmann (2004), which suggests that different mechanisms of inflation transmission across countries in the euro area are the cause of persistent inflation differentials.

5.2 Impulse Responses

In this subsection, we analyze the effects of an innovation to: (i) the euro area common component of the tradable sector technology shock (ε_t^Z), (ii) the Spain-specific component of the tradable sector technology shock ($\varepsilon_t^{Z,T}$),(iii) the Spain-

specific nontradable sector technology shock ($\varepsilon_t^{Z,N}$), (iv) a euro area monetary policy shock (ε_t^m), and (v) a government-spending shock in the nontradable sector in Spain ($\varepsilon_t^{G,N}$). In all cases we present posterior mean impulse responses in the model with raw data and homogeneous Phillips Curves: this does not mean that the responses of inflation will be symmetric across countries, because the composition of the HICP differs across countries (both in the fraction of tradable and nontradable goods, as well as the fraction of imported and domestically produced goods).

Technology Shocks Figures 8-10 display the responses to the three sector-specific technology shocks. In the cases of inflation and growth, the number represents accumulated year-on-year effects. In the case of the real exchange rate and terms of trade, defined as $tot_t = p_t^{F^*} - p_t^H$ (price of imports minus price of exports), we present the evolution of the level. Also, in all cases, the numbers represent deviations from long-term trend values. There are similarities and discrepancies in the reaction of main variables to these shocks. The main similarity is that, in all cases, output growth in Spain and in the euro area increase after a productivity shock. In addition, nontradable inflation in Spain always increases with a tradable sector technology shock: the Balassa-Samuelson effect is present in the estimated model, but its effect is quantitatively small. As a result, the real exchange rate always depreciates under a productivity improvement.

Under a euro area wide tradable sector productivity innovation (Figure 8), HICP inflation declines on impact by 0.14 percent in Spain and by 0.13 percent in the euro area. Nontradable inflation increases but by very small amounts: 0.02 percent. Hence, while the Balassa-Samuelson is present, it is quantitatively small, and the behavior of headline inflation is explained mostly by the behavior of the price of tradable goods. The behavior of inflation does not display much persistence, and after 4 quarters it has returned to its long-term value (of 2 percent). Because of the similar response of headline HICP inflation in Spain and the euro area, the real exchange rate and the terms of trade barely move. Under this shock, growth increases by 0.31 percent in Spain and by 0.35 percent in the euro area on impact, and it exhibits some oscillating behavior (crossing the zero line) before returning to the long-term value. Overall, the effects of a euro area wide productivity shock are symmetric in both Spain and the rest of the euro area.

Under a Spain-only tradable sector technology shock, the effects are more asymmetric (Figure 9). The reaction of Spain variables is stronger, while the reaction of euro

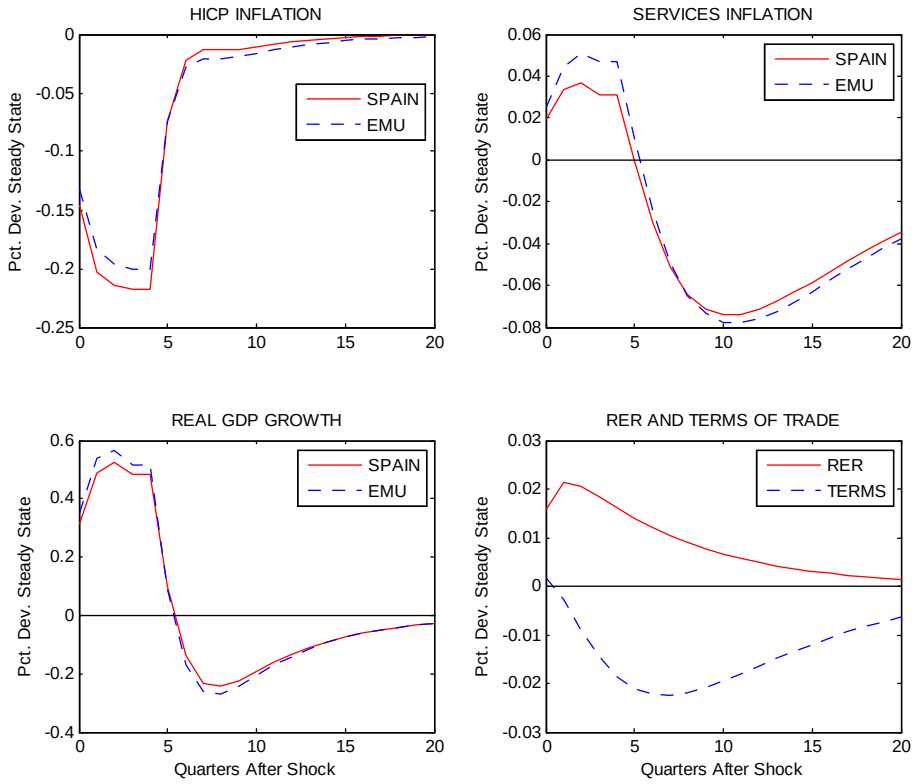

Figure 8: Impulse response to a euro area tradable sector technology shock

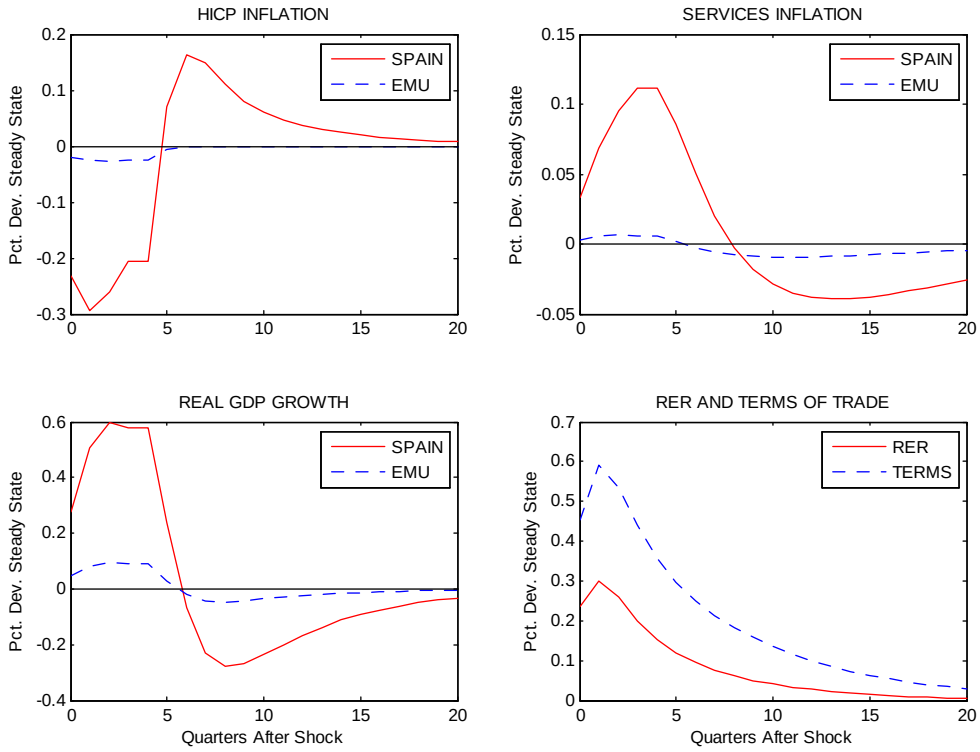

Figure 9: Impulse response to a Spain-only tradable sector technology shock.

area variables is weaker. For the case of Spain, year-on-year inflation decreases by 0.23 percent on impact, and it takes longer for inflation to return to its long-term value. Similarly, output increases on impact by 0.3 percent above trend. Nontradable inflation increases by 0.03 percent, and it displays a hump-shaped response, since it peaks at 0.07 percent after 4 quarters. Again, the Balassa-Samuelson effect is quantitatively small and does not prevent the real exchange rate from depreciating by 0.21 percent on impact and display a hump-shaped response. Because Spain enjoys higher productivity than the rest of the euro area, the terms of trade increase. Even though the shock is asymmetric and only affects the Spanish tradable sector, there are some spillover effects to the rest of the EMU. Since inflation in Spain declines, headline HICP inflation in the EMU declines as well. The ECB cuts rates and this boosts EMU growth to 0.1 percent above trend on impact, with a hump-shaped response.

The effects of a Spain-only nontradable shock are similar to those we have described for the tradable shock, except that the effect is on the nontradable sector (Figure 10). In this case, it is nontradable inflation that declines and displays a hump-shaped

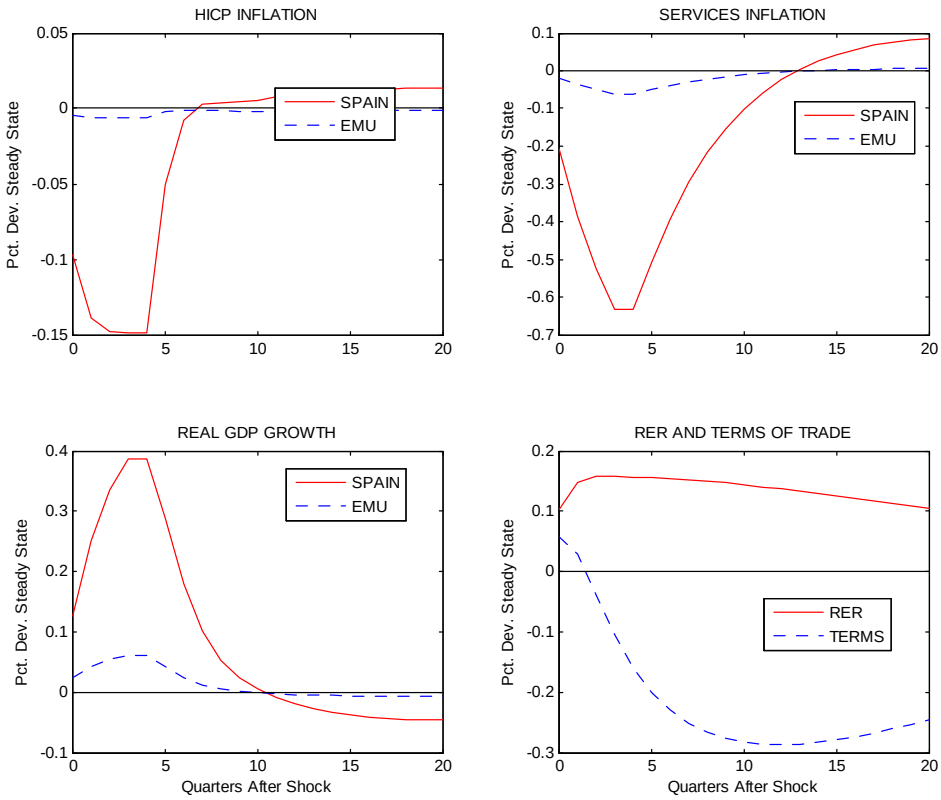

Figure 10: Impulse response to a Spain-only nontradable sector technology shock.

response: the impact is 0.21 percent, and after 4 quarters it is 0.6 percent below trend. As a result, the headline HICP declines, but by a smaller amount, since inflation in the tradable sector increases, and the real exchange rate depreciates. Output growth increases in Spain by 0.14 percent above trend, and displays some hump-shaped response, peaking at 0.4 percent after four quarters. There some small spillover effects to the rest of the EMU, because of the reaction of monetary policy.

Monetary Policy Shocks Figure 11 displays the impulse response to a monetary policy shock that takes decreases the nominal interest rate by 25 basis points at an annualized rate. Similar to the case of euro area productivity shocks, the effects of monetary policy are symmetric in Spain and the euro area. This can be explained because the parameters that reflect preferences and technology across countries are assumed to be the same, while the parameters that are estimated and explain inflation dynamics turn out to be the same as well. Output declines by 0.18 percent below trend after an increase of interest rates, while nontradable inflation

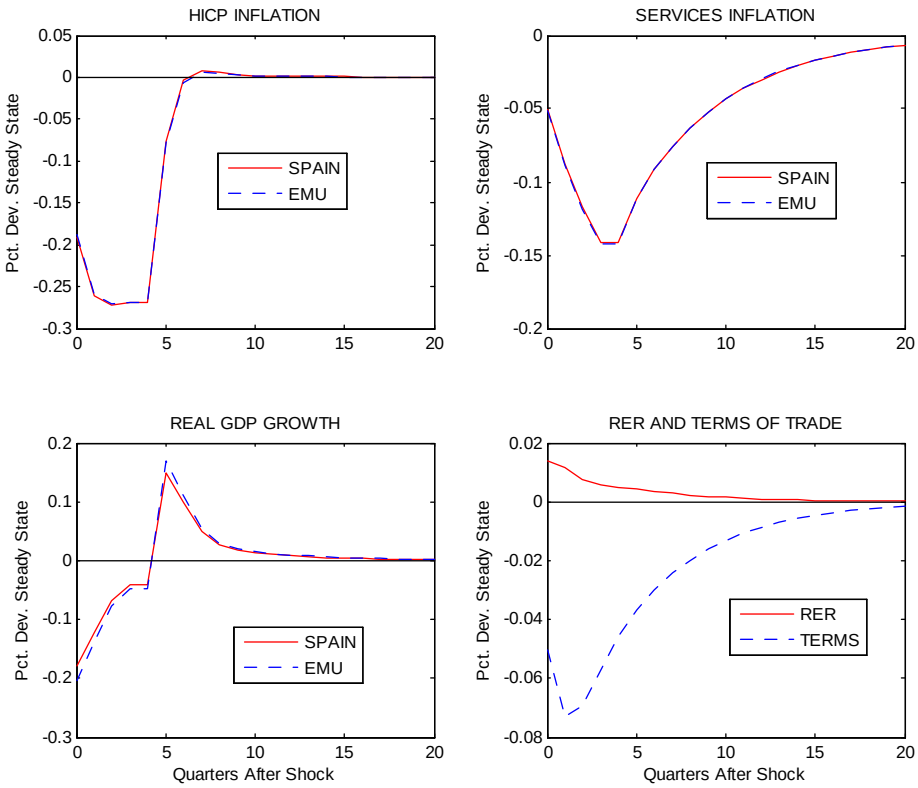

Figure 11: Impulse response to a monetary policy shock.

declines about 0.04 percent below trend on impact, and displays some hump-shaped response. The impact effect of monetary policy on headline HICP inflation is 0.18 percent below trend, which is mostly driven by the jumpy behavior of tradables inflation in both countries. Since the effect is symmetric on both price indices, the real exchange rate does not move.

Response to a Nontradable Demand Shock The response to a nontradable demand shock is presented in Figure 12. The most important result is that output in Spain increases by 0.17 percent above trend on impact. Both nontradable and tradable inflation increase after this type of shock: the nontradable component increases because of excess demand for its product, while the tradable component increases because of the imperfect substitutability of both types of goods: tradable goods producers are able to charge higher prices and not loose market share in the Spanish market. The effects on prices are quatitatively small. In this case, the real exchange rate appreciates, because of higher inflation in both sectors in Spain. Since

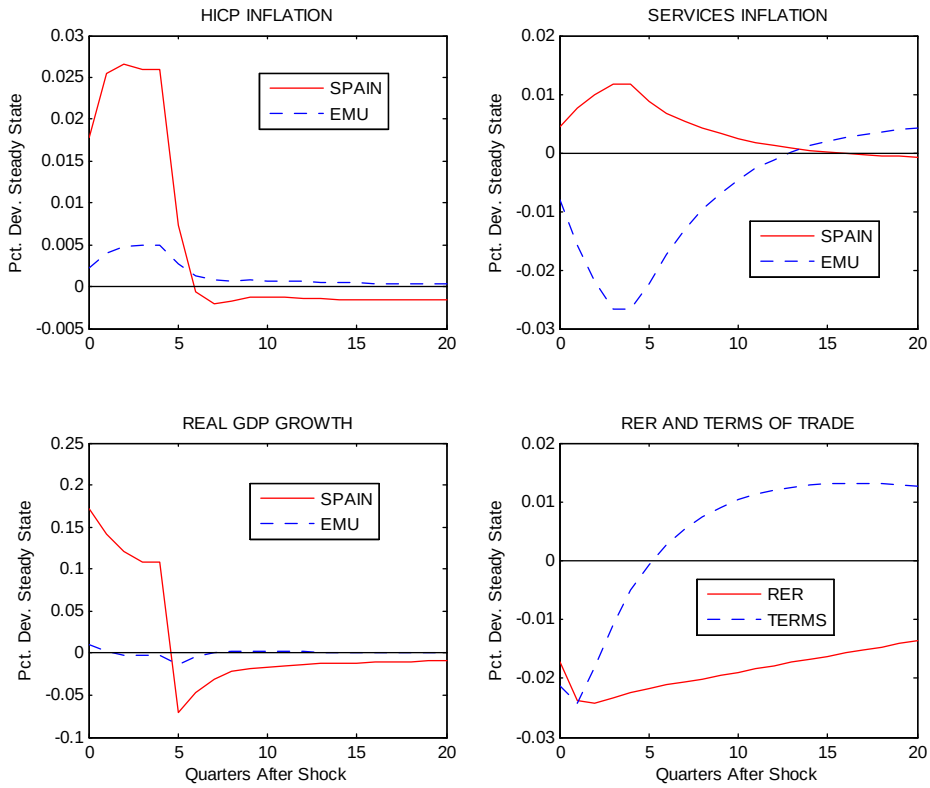

Figure 12: Impulse response to a nontradable government spending shock

this is the only shock that increases nontradable inflation, output, and causes a real appreciation at the same time, López-Salido et al. (2005) suggest that this type of shock would have to be a main ingredient in explaining the behavior of the Spanish economy in the recent years.

5.3 What drives inflation differentials?

What we have learned from the previous subsection is that a combination of shocks would be useful to explain the data: a positive nontradable sector demand shock together with a negative nontraded sector technology shock could explain why inflation in services has been higher than in goods, while at the same time explaining above-trend GDP growth and real exchange rate appreciation. To better understand what forces are behind the behavior of main macro variables in the EMU and in Spain, we perform a variance decomposition exercise (Table 5).[18]

[18] The variance decomposition is performed using the posterior mean of the model estimated with raw data and homogeneous Phillips curves. The results under the other estimations are available

Table 5. Variance decomposition (in percent)

	Δp^{EMU}	$\Delta p^{N,EMU}$	Δy^{EMU}	Δp	Δp^{N}	Δy	Δy^{N}	r	$\Delta p^{*} - \Delta p$
ε^{m}	53.8	7.7	5.0	27.0	4.5	3.8	5.6	14.3	0.02
ε^{x}	5.2	0.1	47.5	3.2	0.03	52.9	39.0	2.7	0.1
ε^{Z}	27.2	5.6	18.4	16.2	2.8	15.0	10.0	53.6	0.4
$\varepsilon^{Z,T}$	3.2	9.5	2.4	40.8	2.8	18.0	11.3	6.5	61.8
$\varepsilon^{Z,N}$	0.3	7.83	0.5	7.2	88.4	5.0	9.5	0.8	12.5
$\varepsilon^{G,T}$	0.01	0.2	0.01	0.1	0.02	1.2	0.0	0.01	0.2
$\varepsilon^{G,N}$	0.01	0.3	0.02	0.3	0.03	3.0	24.5	0.03	0.4
$\varepsilon^{Z,T^{*}}$	9.2	1.3	6.0	2.0	0.7	0.6	0.0	17.9	16.9
$\varepsilon^{Z,N^{*}}$	0.6	63.3	1.6	2.1	0.4	0.2	0.0	2.7	4.9
$\varepsilon^{G,T^{*}}$	0.4	4.2	18.2	1.0	0.2	0.2	0.0	1.3	3.3
$\varepsilon^{G,N^{*}}$	0.01	0.4	0.4	0.02	0.01	0.01	0.0	0.04	0.07

Several interesting results arise. First, euro area variables are mostly explained by euro area shocks, specially euro area inflation, which is mostly driven by monetary policy shocks, and euro area growth, which is driven by neutral and tradable sector technology shocks. About 80 percent of the volatility of nominal interest rates is driven by technology shocks that affect the tradable sector, and an additional 14.3 percent is driven by.monetary policy shocks. Second, nontradable inflation both in Spain and the euro area is mostly driven by nontradable technology shocks, while tradable sector technology shocks have a small impact, explaining about 16 percent of nontradable inflation volatility in the euro area and 6 percent in Spain. Therefore, this confirms that while the impulse responses show that there is indeed a Balassa-Samuelson effect, this turns out to be quantitatively unimportant. Third, government spending shocks turn out to be insignificant to explain other variables than output growth. They explain about one quarter of the volatility of nontradable output growth in Spain, and about one fifth of the volatility of output growth in the euro area. Indeed, most output growth in Spain and in the EMU are explained by common technology shocks in the euro area (neutral technology, monetary and euro area wide tradable sector productivity shocks).

Most importantly, the main result of Table 5 is that most of the volatility in the

upon request, and the main qualitative results do not change.

inflation differential turns out to be explained by tradable sector technology shocks: their contribution is 78 percent of the variance of total volatility. Nontradable sector shocks explain 17.5 percent, and the rest of the shocks have marginal importance. These results are in contrast with the findings of Altissimo et al. (2005), who suggest that nontradable productivity shocks are a main driver of inflation differentials in the euro area. They base their explanation on overall inflation dispersion in the euro area and using evidence similar to Figure 1, where services inflation seems to be main driver of HICP inflation. In the present paper, as we have shown in Figure 4, differentials in the tradable goods sector inflation are the main driver of HICP inflation differentials between Spain and the EMU. Therefore, it could well be that explaining inflation differentials country by country would deliver different results than the Spanish case. It is important to remark that our results are similar to those of Duarte and Wolman (2002): their paper also finds that shocks to the tradable sector are a main driver of inflation differentials. Finally, as in the present paper, both Altissimo et al. (2005) and Duarte and Wolman (2002) find a negligible effect of fiscal or demand shocks on inflation differentials.

6 Conclusions

The study of inflation differentials in a currency union has become important, specially after the observed increase in inflation dispersion and the persistence of inflation differentials in the euro area after the launch of the euro in January 1999. Several explanations have been suggested in the literature, that emphasize the role of tradable sector and nontradable sector technology shocks, demand shocks, and heterogeneous inflationary processes in the euro area. This paper has contrasted all these hypotheses for the case of Spain, in a two-sector, two-country DSGE model estimated with Bayesian methods. An obvious shortcoming of the paper is the short sample used to estimate the model, but the process of nominal convergence between Spain and the rest of the EMU countries is too important to be ignored, and the pre-EMU sample cannot be used in a model where coefficients are not time-varying.

The results can be summarized as follows: first, the estimated degrees of nominal rigidity across countries and sectors are similar to those obtained with survey evidence by Fabiani et al. (2006). Second, when the estimation is conducted allowing for effects of assuming positive inflation rates, these turn out to be unimportant for

the structural parameter estimates, but increase the autocorrelation of the shocks significantly. Third, we cannot reject the hypothesis that inflation dynamics in Spain and the rest of the euro area are similar. Still, the impact of symmetric shocks can be different due to the different composition of each country's CPI. Finally, the most important explanation for the inflation differential between Spain and the euro area comes from tradable sector productivity shocks that affect either Spain, the rest of the euro area, and both. On the other hand, nontradable technology shocks have a minor contribution to explaining inflation differentials. Demand shocks are useful to explain a fraction of output growth volatility but not of inflation dispersion.

Some caveats might apply to our results. First of all, the effects of price markup shocks (that would increase the market power of firms) and productivity shocks cannot be distinguished in the context of this model. Therefore, what we are attributing as productivity shocks in the tradable sector could be attributed to time-varying markups, and hence the results we provide here can be seen as an upper bound to the importance of technology shocks. Note, however, that this is simply a labelling issue, and would not change the fact that the bulk of the action to explain the inflation differential between Spain and the rest of the EMU is in the tradable sector. Second, it could well be that the importance of the tradable sector productivity shock is picking up the effect of oil price shocks, that are not included in the model. Finally, while the EMU is the most important trade partner of Spain (60 percent of international trade), the role of trade with third countries, the role of other commodity prices and the effects of the trade-weighted euro exchange rate should be introduced in large scale macroeconomic models.

A very important extension to this paper would be to estimate the model with time-varying inflation targets, and examine the implications for the estimated model's parameters, taking into account the (time-varying) role of steady-state inflation. Newly developed tecniques in nonlinear estimation of DSGE models using particle filters (Fernández-Villaverde and Rubio-Ramírez, 2006; and Amisano and Tristani, 2006) could be implemented in a large scale model as the one presented here. We leave this important computational challenge for future research.

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A Appendix: Steady-State Effects of a Positive Inflation Rate

In this section we focus on inflation dynamics in the home country. The same analysis applies to the other country. In the steady-state, we have that:

$$\frac{\hat{p}^N}{P^N} K^{N,1} = \frac{\sigma}{(\sigma - 1)} K^{N,2}$$

where

$$\left[1 - \beta\theta_N \left(\Pi^N\right)^{-(1-\varphi_N)(1-\sigma)}\right] K^{N,1} = \tilde{\lambda}\tilde{Y}^N$$

$$\left[1 - \beta\theta_N \left(\Pi^N\right)^{(1-\varphi_N)\sigma}\right] K^{N,2} = \tilde{\lambda}\frac{MC^N}{P^N}\tilde{Y}^N$$

Therefore:

$$\frac{\hat{p}^N}{P^N} = \frac{\sigma}{(\sigma - 1)} \frac{MC^N}{P^N} \frac{\left[1 - \beta\theta_N \left(\Pi^N\right)^{-(1-\varphi_N)(1-\sigma)}\right]}{\left[1 - \beta\theta_N \left(\Pi^N\right)^{(1-\varphi_N)\sigma}\right]}$$

From the evolution of prices under Calvo pricing:

$$\left(\frac{\hat{p}^N}{P^N}\right)^{1-\sigma} = \frac{1 - \theta_N \left(\Pi^N\right)^{-(1-\varphi_N)(1-\sigma)}}{(1 - \theta_N)}$$

Note that if gross steady-state inflation is one ($\Pi^N = 1$), or there is full indexation ($\varphi_N = 1$), then the usual condition applies and prices are a markup over marginal costs. But if there is trend inflation, and partial indexation, then there will be price dispersion in equilibrium between those who reset prices optimally and those who do not. Therefore:

$$\frac{MC^N}{P^N} = \frac{(\sigma - 1)}{\sigma} \frac{\left[1 - \beta\theta_N \left(\Pi^N\right)^{(1-\varphi_N)\sigma}\right]}{\left[1 - \beta\theta_N \left(\Pi^N\right)^{-(1-\varphi_N)(1-\sigma)}\right]} \left[\frac{1 - \theta_N \left(\Pi^N\right)^{-(1-\varphi_N)(1-\sigma)}}{(1 - \theta_N)}\right]^{\frac{1}{1-\sigma}}$$

Similarly, for the tradable sector:

$$\frac{MC^H}{P^H} = \frac{(\sigma - 1)}{\sigma} \frac{\left[1 - \beta\theta_H \left(\Pi^H\right)^{(1-\varphi_H)\sigma}\right]}{\left[1 - \beta\theta_H \left(\Pi^H\right)^{-(1-\varphi_H)(1-\sigma)}\right]} \left[\frac{1 - \theta_H \left(\Pi^H\right)^{-(1-\varphi_H)(1-\sigma)}}{(1 - \theta_H)}\right]^{\frac{1}{1-\sigma}}$$

From the evolution of prices under Calvo pricing:

$$\left(\frac{\hat{p}^H}{P^H}\right)^{1-\sigma} = \frac{1-\theta_H\left(\Pi^H\right)^{-(1-\varphi_H)(1-\sigma)}}{(1-\theta_H)}$$

Since real wages are the same in the two sectors, then :

$$\frac{MC^N}{P^N} = \frac{\tilde{W}}{Z^N}\frac{P}{P^N}$$

$$\frac{MC^H}{P^H} = \frac{\tilde{W}}{Z^T}\frac{P}{P^H}$$

Now, we work backwards to ensure that all prices are the same in steady-state, despite sectoral differences in technology, and degrees of nominal stickiness. Dividing the previous expressions:

$$\frac{P^H}{P^N}\frac{MC^N}{MC^H} = \frac{\frac{\left[1-\beta\theta_N\left(\Pi^N\right)^{(1-\varphi_N)\sigma}\right]}{\left[1-\beta\theta_N(\Pi^N)^{-(1-\varphi_N)(1-\sigma)}\right]}\left[\frac{1-\theta_N\left(\Pi^N\right)^{-(1-\varphi_N)(1-\sigma)}}{(1-\theta_N)}\right]^{\frac{1}{1-\sigma}}}{\frac{\left[1-\beta\theta_H(\Pi^H)^{(1-\varphi_H)\sigma}\right]}{\left[1-\beta\theta_H(\Pi^H)^{-(1-\varphi_H)(1-\sigma)}\right]}\left[\frac{1-\theta_H(\Pi^H)^{-(1-\varphi_H)(1-\sigma)}}{(1-\theta_H)}\right]^{\frac{1}{1-\sigma}}} = \Phi$$

We assume that $P^H = P^N = P$. Then, we can calibrate $\Phi = \frac{Z^T}{Z^N}$, such that price levels are the same across sectors, while marginal costs and markups are not. Another possibility is to consider that all the coefficients regarding nominal rigidities are the same, then $\Phi = 1$, and marginal costs and prices would be the same across sectors. Because one of the main points of this paper is to look at heterogeneity in Phillips curves, assuming the same Calvo lotteries and indexation parameters would not allow for this channel.

B Appendix: Loglinear approximation

Here we present the linearized model that we estimate in the paper. Consumption and production levels, and real wages are normalized by the level of technology to make them stationary. Lower case variables denote percent deviations from steady state (i.e. $l_t = \log(L_t) - \log(L) \approx \frac{L_t - L}{L}$), while lower case variables with a tilde denote percent deviations from steady state for those variables normalized by the level of technology (i.e. $\tilde{c}_t = \log(\tilde{C}_t) - \log(\tilde{C})$, where $\tilde{C}_t = \frac{C_t}{X_t}$).

B.1 Euler equation and risk sharing

$$\tilde{\lambda}_t = E_t\tilde{\lambda}_{t+1} + (r_t - E_t\Delta p_{t+1}) \tag{32}$$

$$\tilde{\lambda}_t = \frac{\tilde{c}_t - \tilde{b}c_{t-1}}{(1-\frac{b}{1+x})} - \frac{b}{(1-\frac{b}{1+x})}\varepsilon_t^x - \xi_t \tag{33}$$

$$\tilde{\lambda}_t^* = \frac{\tilde{c}_t^* - \tilde{b}^*c_{t-1}^*}{(1-\frac{b^*}{1+x})} - \frac{b^*}{(1-\frac{b^*}{1+x})}\varepsilon_t^x - \xi_t^* \tag{34}$$

$$rer_t = \lambda_t^* - \lambda_t \tag{35}$$

B.2 Demand functions

Let´s define the following relative prices: $t_t^N = p_t^N - p_t$, $t_t^H = p_t^H - p_t$, $t_t^F = p_t^F - p_t$, $t_t^{N^*} = p_t^{N^*} - p_t^*$, $t_t^{H^*} = p_t^{H^*} - p_t^*$, $t_t^{F^*} = p_t^{F^*} - p_t^*$. Then, the consumption demand functions by households are:

$$\tilde{c}_t^H = -[\varepsilon(1-\gamma)\lambda + \theta(1-\lambda)]t_t^H - (1-\lambda)[\varepsilon(1-\gamma) - \theta]t_t^F + \varepsilon(1-\gamma)t_t^N + \tilde{c}_t \tag{36}$$

$$\tilde{c}_t^F = -\lambda[\varepsilon(1-\gamma) - \theta]t_t^H - [\varepsilon(1-\gamma)(1-\lambda) + \theta\lambda]t_t^F + \varepsilon(1-\gamma)t_t^N + \tilde{c}_t \tag{37}$$

$$\tilde{c}_t^N = -\varepsilon t_t^N + \tilde{c}_t \tag{38}$$

$$\tilde{c}_t^{H^*} = -[\varepsilon(1-\gamma^*)(1-\lambda^*) + \theta\lambda^*]t_t^{H^*} - \lambda^*[\varepsilon(1-\gamma^*) - \theta]t_t^{F^*} + \varepsilon(1-\gamma^*)t_t^{N^*} + \tilde{c}_t^* \tag{39}$$

$$\tilde{c}_t^{F^*} = -\varepsilon(1-\lambda^*)[(1-\gamma^*) - \theta]t_t^{H^*} - [\varepsilon(1-\gamma^*)\lambda^* + \theta(1-\lambda^*)]t_t^{F^*} + \varepsilon(1-\gamma^*)t_t^{N^*} + \tilde{c}_t^* \tag{40}$$

$$\tilde{c}_t^{N^*} = -\varepsilon t_t^{N^*} + \tilde{c}_t^* \tag{41}$$

B.3 Labor supply

$$\left(1 - \frac{b}{1+x}\right)(\xi_t + \tilde{w}_t - \phi l_t) = \tilde{c}_t - \frac{b}{1+x}(\tilde{c}_{t-1} - \varepsilon_t^x) \tag{42}$$

$$\left(1 - \frac{b^*}{1+x}\right)(\xi_t^* + \tilde{w}_t^* - \phi^* l_t^*) = \tilde{c}_t^* - \frac{b^*}{1+x}\left(\tilde{c}_{t-1}^* - \varepsilon_t^x\right) \tag{43}$$

The hours allocation across sectors is:

$$l_t = (1-a_k)l_t^N + a_k l_t^T \tag{44}$$

$$l_t^* = (1-a_{k^*})l_t^{N^*} + a_{k^*}l_t^{T^*} \tag{45}$$

where $a_k = \frac{\gamma}{\gamma+(1-\gamma)Z^T}$, and $a_k^* = \frac{\gamma^*}{\gamma^*+(1-\gamma^*)Z^T}$. In the model with zero inflation in the steady-state, $Z^T = 1$, and $a_k = \gamma$, $a_k^* = \gamma^*$.

B.4 Technology

The production functions in the two sectors are:

$$\tilde{y}_t^N = z_t^N + l_t^N \tag{46}$$

and

$$\tilde{y}_t^T = z_t^T + l_t^T \tag{47}$$

and

$$\tilde{y}_t^{N^*} = z_t^{N^*} + l_t^{N^*} \tag{48}$$

and

$$\tilde{y}_t^{T^*} = z_t^{T^*} + l_t^{T^*} \tag{49}$$

B.5 Price setting

Define $(\Pi_t^*)^N = \frac{\hat{p}_t^N}{P_t^N}$. Then

$$(\Pi_t^*)^N + k_t^{N,1} = k_t^{N,2} \tag{50}$$

where

$$k_t^{N,1} = [1-\beta\theta_N\Pi^{-(1-\varphi_N)(1-\sigma)}](\tilde{\lambda}_t+\tilde{y}_t^N)+\beta\theta_N\Pi^{-(1-\varphi_N)(1-\sigma)}[k_{t+1}^{N,1}-(1-\sigma)(\Delta p_{t+1}^N-\varphi_N\Delta p_t^N)] \tag{51}$$

$$k_t^{N,2} = [1-\beta\theta_N\Pi^{(1-\varphi_N)\sigma)}](\tilde{\lambda}_t+\tilde{y}_t^N+mc_t^N)+\beta\theta_N\Pi^{(1-\varphi_N)\sigma}[k_{t+1}^{N,2}+\sigma(\Delta p_{t+1}^N-\varphi_N\Delta p_t^N)] \tag{52}$$

The evolution of the price level of home non-tradables is

$$(\Pi_t^*)^N = \frac{\theta_N\Pi^{-(1-\varphi_N)(1-\sigma)}}{(1-\theta_N)((\Pi^*)^N)^{1-\sigma}}(\Delta p_t^N - \varphi_N\Delta p_{t-1}^N) \tag{53}$$

Define $mc_t^N = \log(\frac{MC_t^N}{P_t^N}) - \log(\frac{MC^N}{P^N})$, and $t_t^N = p_t^N - p_t$. Then:

$$mc_t^N = \tilde{w}_t - z_t^N - t_t^N \tag{54}$$

Define $(\Pi_t^*)^H = \frac{\hat{p}_t^H}{P_t^H}$. Then

$$(\Pi_t^*)^H + k_t^{H,1} = k_t^{H,2} \tag{55}$$

where

$$k_t^{H,1} = [1-\beta\theta_H\Pi^{-(1-\varphi_H)(1-\sigma)}](\tilde{\lambda}_t+\tilde{y}_t^H)+\beta\theta_H\Pi^{-(1-\varphi_H)(1-\sigma)}[k_{t+1}^{H,1}-(1-\sigma)(\Delta p_{t+1}^H-\varphi_H\Delta p_t^H] \tag{56}$$

$$k_t^{H,2} = [1-\beta\theta_H\Pi^{(1-\varphi_H)\sigma)}](\tilde{\lambda}_t+\tilde{y}_t^H+mc_t^H)+\beta\theta_H\Pi^{(1-\varphi_H)\sigma}[k_{t+1}^{H,2}+\sigma(\Delta p_{t+1}^H-\varphi_H\Delta p_t^H] \tag{57}$$

The evolution of the price level of home tradables is

$$(\Pi_t^*)^H = \frac{\theta_H\Pi^{-(1-\varphi_H)(1-\sigma)}}{(1-\theta_H)((\Pi^*)^H)^{1-\sigma}}(\Delta p_t^H - \varphi_H\Delta p_{t-1}^H) \tag{58}$$

Define $mc_t^H = \log(\frac{MC_t^H}{P_t^H}) - \log(\frac{MC^H}{P^H})$, and $t_t^H = p_t^H - p_t$.Then:

$$mc_t^H = \tilde{w}_t - z_t^T - t_t^H \tag{59}$$

Define $(\Pi_t^*)^{N^*} = \frac{\hat{p}_t^{N^*}}{P_t^{N^*}}$. Then

$$(\Pi_t^*)^{N^*} + k_t^{N^*,1} = k_t^{N^*,2} \tag{60}$$

where

$$k_t^{N^*,1} = [1-\beta\theta_{N^*}\Pi^{-(1-\varphi_{N^*})(1-\sigma)}](\lambda_t+\tilde{y}_t^{N^*})+\beta\theta_{N^*}\Pi^{-(1-\varphi_{N^*})(1-\sigma)}[k_{t+1}^{N^*,1}-(1-\sigma)(\Delta p_{t+1}^{N^*}-\varphi_{N^*}\Delta p_t^{N^*})] \tag{61}$$

$$k_t^{N^*,2} = [1-\beta\theta_{N^*}\Pi^{(1-\varphi_{N^*})\sigma)}](\lambda_t+\tilde{y}_t^{N^*}+mc_t^{N^*})+\beta\theta_{N^*}\Pi^{(1-\varphi_{N^*})\sigma}[k_{t+1}^{N^*,2}+\sigma(\Delta p_{t+1}^{N^*}-\varphi_{N^*}\Delta p_t^{N^*})] \tag{62}$$

The evolution of the price level of foreign non-tradables is

$$(\Pi_t^*)^{N^*} = \frac{\theta_{N^*}\Pi^{-(1-\varphi_{N^*})(1-\sigma)}}{(1-\theta_{N^*})((\Pi^*)^{N^*})^{1-\sigma}}(\Delta p_t^{N^*} - \varphi_{N^*}\Delta p_{t-1}^{N^*}) \tag{63}$$

Define $mc_t^{N^*} = \log(\frac{MC_t^{N^*}}{P_t^{N^*}}) - \log(\frac{MC^{N^*}}{P^{N^*}})$, and $t_t^{N^*} = p_t^{N^*} - p_t^*$. Then:

$$mc_t^{N^*} = \tilde{w}_t - z_t^{N^*} - t_t^{N^*} \tag{64}$$

Define $(\Pi_t^*)^{F^*} = \frac{\hat{p}_t^{F^*}}{P_t^{F^*}}$. Then

$$(\Pi_t^*)^{F^*} + k_t^{F^*,1} = k_t^{F^*,2} \tag{65}$$

where

$$k_t^{F^*,1} = [1-\beta\theta_{F^*}\Pi^{-(1-\varphi_{F^*})(1-\sigma)}](\lambda_t+\tilde{y}_t^{F^*})+\beta\theta_{F^*}\Pi^{-(1-\varphi_{F^*})(1-\sigma)}[k_{t+1}^{F^*,1}-(1-\sigma)(\Delta p_{t+1}^{F^*}-\varphi_{N^*}\Delta p_t^{F^*})] \tag{66}$$

$$k_t^{F^*,2} = [1-\beta\theta_{F^*}\Pi^{(1-\varphi_{F^*})\sigma)}](\lambda_t+\tilde{y}_t^{F^*}+mc_t^{F^*})+\beta\theta_{F^*}\Pi^{(1-\varphi_{F^*})\sigma}[k_{t+1}^{F^*,2}+\sigma(\Delta p_{t+1}^{F^*}-\varphi_{F^*}\Delta p_t^{F^*})] \tag{67}$$

The evolution of the price level of foreign tradables is

$$(\Pi_t^*)^{F^*} = \frac{\theta_{F^*}\Pi^{-(1-\varphi_{F^*})(1-\sigma)}}{(1-\theta_{F^*})((\Pi^*)^{F^*})^{1-\sigma}}(\Pi_t^{F^*} - \varphi_{F^*}\Pi_{t-1}^{F^*}) \tag{68}$$

Define $mc_t^{F^*} = \log(\frac{MC_t^{F^*}}{P_t^{F^*}}) - \log(\frac{MC^{F^*}}{P^{F^*}})$, and $t_t^{F^*} = p_t^{F^*} - p_t^*$. Then:

$$mc_t^{F^*} = \tilde{w}_t - z_t^{T^*} - t_t^{F^*} \tag{69}$$

B.6 Relevant price indices

The Spanish consumer price inflation is given by

$$\Delta p_t = \gamma\lambda\Delta p_t^H + \gamma(1-\lambda)\Delta p_t^{F^*} + (1-\gamma)p_t^N \tag{70}$$

The rest of euro area aggregates are given by:

$$\Delta p_t^* = \gamma^*(1-\lambda^*)\Delta p_t^H + \gamma^*\lambda^*\Delta p_t^{F^*} + (1-\gamma^*)p_t^N \tag{71}$$

The European Central Bank targets the euro-area wide CPI, which is assumed to be an harmonic mean of the price levels of Spain and the rest of the euro area:

$$\Delta p_t^{EU} = s\Delta p_t + (1-s)\Delta p_t^* \tag{72}$$

using the country sizes as weights.

The relative price variables move as follows:

$$t_t^N = t_{t-1}^N + \Delta p_t^N - \Delta p_t \tag{73}$$

$$t_t^H = t_{t-1}^H + \Delta p_t^H - \Delta p_t \tag{74}$$

$$t_t^F = t_{t-1}^F + \Delta p_t^F - \Delta p_t \tag{75}$$

$$t_t^{N^*} = t_{t-1}^{N^*} + \Delta p_t^{N^*} - \Delta p_t^* \tag{76}$$

$$t_t^{F^*} = t_{t-1}^{F^*} + \Delta p_t^{F^*} - \Delta p_t^* \tag{77}$$

$$t_t^{H^*} = t_{t-1}^{H^*} + \Delta p_t^{H^*} - \Delta p_t^* \tag{78}$$

and the real exchange rate moves as:

$$rer_t = rer_{t-1} + \Delta p_t^* - \Delta p_t \tag{79}$$

B.7 Market Clearing

The market clearing for the nontradable goods sector at home is

$$\tilde{y}_t^N = (1-\eta)\tilde{c}_t^N + \eta\tilde{g}_t^N \tag{80}$$

The market clearing condition in the tradable goods sector at home is

$$\tilde{y}_t^H = (1-\eta)[\lambda\tilde{c}_t^H + (1-\lambda)\tilde{c}_t^{H^*}] + \eta\tilde{g}_t^T \tag{81}$$

For the foreign country, these expressions become:

$$\tilde{y}_t^{N^*} = (1-\eta^*)c_t^{N^*} + \eta^* g_t^{N^*} \tag{82}$$

and

$$\tilde{y}_t^{F^*} = (1-\eta^*)[(1-\lambda^*)\tilde{c}_t^F + \lambda^*\tilde{c}_t^{F^*}] + \eta^*\tilde{g}_t^{T^*} \tag{83}$$

Aggregate real GDP aggregates traded and nontraded goods using the appropiate relative prices:

$$\tilde{y}_t = \gamma(t_t^H + \tilde{y}_t^H) + (1-\gamma)(t_t^N + \tilde{y}_t^N) \tag{84}$$

$$\tilde{y}_t^* = \gamma^*(t_t^{F^*} + y_t^{F^*}) + (1-\gamma^*)(t_t^{N^*} + y_t^{N^*}) \tag{85}$$

B.8 Monetary Policy

In order to abstract from fiscal policy considerations, it is assumed that government spending in the two areas is financed through lump sum taxes. Monetary policy is conducted by the ECB with a Taylor rule that only targets the EMU CPI:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\gamma_\pi \Delta p_t^{EMU} + \varepsilon_t^m \tag{86}$$

where z_t is an iid monetary policy shock.

C Appendix: Posterior Estimates

Table A. Posterior Distributions

	Heterogeneous		*Homogeneous*	
	Raw Data	*Demeaned*	*Raw Data*	*Demeaned*
$\rho^{Z,T}$	0.83 (0.78- 0.88)	0.67 (0.57- 0.76)	0.83 (0.78- 0.88)	0.67 (0.58- 0.76)
$\rho^{G,T}$	0.97 (0.97- 0.98)	0.69 (0.57- 0.82)	0.97 (0.96- 0.99)	0.70 (0.58- 0.82)
$\rho^{Z,N}$	0.98 (0.98 - 0.99)	0.77 (0.68 - 0.86)	0.96 (0.94 - 0.98)	0.77 (0.68 - 0.86)
$\rho^{G,N}$	0.96 (0.94 - 0.98)	0.70 (0.56 - 0.84)	0.96 (0.94 - 0.98)	0.71 (0.57 - 0.84)
$\sigma(\varepsilon_t^x)$	1.29 (1.02 - 1.49)	0.86 (0.68 - 1.03)	1.23 (1.01 - 1.46)	0.86 (0.67 - 1.04)
$\sigma(\varepsilon_t^m)$	0.16 (0.11 - 0.21)	0.23 (0.17 - 0.3)	0.17 (0.11 - 0.22)	0.24 (0.17 - 0.31)
$\sigma(\varepsilon_t^Z)$	1.19 (0.77 - 1.62)	1.08 (0.76 - 1.41)	1.23 (0.84 - 1.63)	1.06 (0.76 - 1.36)
$\sigma(\varepsilon_t^{Z,T})$	1.55 (1.05 - 2.02)	1.14 (0.79 - 1.49)	1.59 (1.13 - 2.01)	1.19 (0.87 - 1.51)
$\sigma(\varepsilon_t^{Z,T^*})$	0.96 (0.46 - 1.38)	0.64 (0.32 - 0.96)	0.97 (0.56 - 1.35)	0.61 (0.32 - 0.91)
$\sigma(\varepsilon_t^{Z,N})$	1.65 (1.45 - 1.91)	1.29 (0.91 - 1.67)	2.31 (1.69 - 2.85)	1.71 (1.21 - 2.07)
$\sigma(\varepsilon_t^{Z,N^*})$	1.46 (1.07 - 1.85)	1.22 (0.84 - 1.61)	1.27 (0.98 - 1.56)	0.92 (0.66 - 1.18)
$\sigma(\varepsilon_t^{G,T})$	1.11 (0.43 - 1.74)	1.79 (1.31 - 2.24)	1.09 (0.39 - 1.75)	1.76 (1.31 - 2.23)
$\sigma(\varepsilon_t^{G,T^*})$	4.60 (3.82 - 5.32)	2.21 (1.27 - 3.08)	4.68 (3.81 - 5.56)	2.31 (1.60 - 3.14)
$\sigma(\varepsilon_t^{G,N})$	3.16 (2.59 - 3.75)	1.97 (1.55 - 2.41)	3.18 (2.57 - 3.71)	1.98 (1.53 - 2.41)
$\sigma(\varepsilon_t^{G,N^*})$	1.11 (0.22 - 1.94)	1.46 (0.22 - 2.78)	1.11 (0.20 - 1.93)	1.37 (0.24 - 2.57)